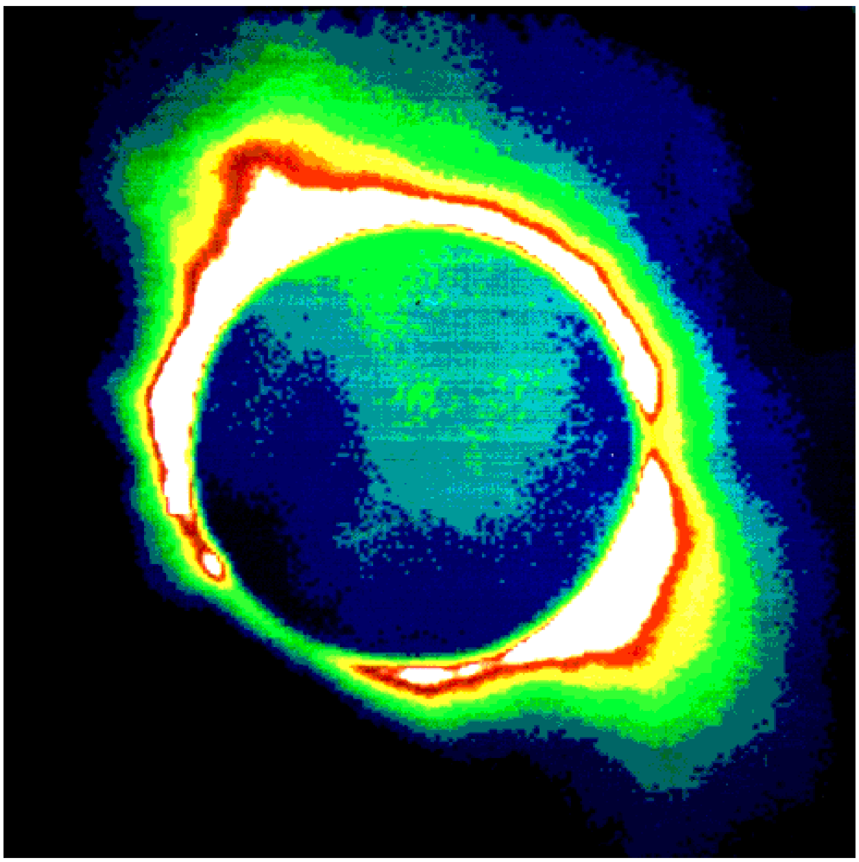
HANDS-ON UNIVERSE

HIGH SCHOOL SCIENCE AND MATH
IN THE CONTEXT OF ASTRONOMY
INVESTIGATIONS

Measuring Size



by Lawrence Hall of Science
University of California, Berkeley
Lawrence Berkeley National Laboratory
and TERC of Cambridge, Massachusetts

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HOU provides a visual and analytic way of exploring the universe.

Use HOU images from professional telescopes, along with HOU image processing software, to pursue investigations of astronomical objects, phenomena, and concepts. Opportunities available to HOU students can lead to accessing professional-grade telescopes via the World-Wide Web for observations as part of research projects such as searching for supernovae and asteroids.

- HOU has been developed and operated by staff at Lawrence Berkeley National Laboratory and Space Science Laboratories at the University of California at Berkeley, Californa, with generous support from the National Science Foundation (grant # ESI-9252915) and the US Department of Energy. The HOU curriculum was developed by TERC with contributions from the Berkeley staff and teachers. The educational center for HOU currently resides at Lawrence Hall of Science, University of California, Berkeley.

- The Berkeley staff responsible for the development of the HOU system includes:
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- Lawrence Hall of Science HOU staff includes:
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Measuring Size

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Constants Sheet

Planetary Data:

Planet	Mass (kg)	Ave Radius (m)	Ave Orbital Radius(m)
Mercury	3.32×10^{23}	2.44×10^{6}	5.79×10^{10}
Venus	4.87×10^{24}	6.08×10^{6}	1.08×10^{11}
Earth	5.97×10^{24}	6.36×10^{6}	1.49×10^{11}
(Moon)	7.35×10^{22}	1.74×10^{6}	
Mars	6.42×10^{23}	3.40×10^{6}	2.28×10^{11}
Jupiter	1.90×10^{27}	6.80×10^{7}	7.78×10^{11}
Saturn	5.69×10^{26}	5.70×10^{7}	1.43×10^{12}
Uranus	8.69×10^{25}	2.51×10^{7}	2.87×10^{12}
Neptune	1.03×10^{26}	2.44×10^{6}	4.50×10^{12}
Pluto	1.30×10^{22}	1.50×10^{6}	5.90×10^{12}

Physical and Astronomical Constants:

Gravitational Constant = G = 6.673×10^{-11} N-m^2/kg^2
Speed of Light in a vacuum = c = 2.9979×10^{8} m/s
Earth-Sun Distance = Astronomical Unit = AU = 1.496×10^{11} m
Earth-Moon Distance = 3.844×10^{8} m
Parsec = pc = 206265 AU = 3.26 ly = 3.09×10^{16} m
Light year = ly = 9.5×10^{15} m
Mass of the Sun = 1.989×10^{30} kg
Luminosity of the Sun = 3.83×10^{26} W
Radius of the Sun = 6.96×10^{8} m

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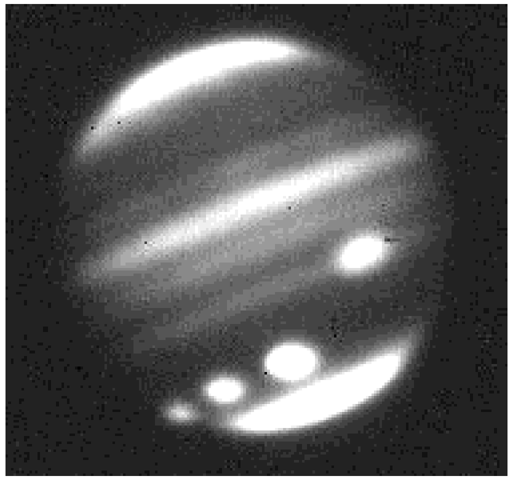
JUPITER CRASH UNIT



• **Open** the image *Fireball.* This is not the same as the image shown in the picture above.

Screen Setup: *Fireball.*

• If the image opens upside down (as it does originally), go to the **Manipulation** menu and select **Flip** and click on **Vertical** and then **OK**.

The fireball was made by a piece of the comet Shoemaker-Levy 9 in July, 1994. The piece was called Fragment G, and it was probably about two miles (three or four kilometers) across. It hit on the back side of Jupiter, where no one could see it, but Jupiter's very fast rotation brought it into view in only a few minutes. Jupiter, even though it is much bigger than Earth, turns faster on its axis, about once every ten hours.

The fireball was visible for only minutes and was easier to see in the infra red, wavelengths of light longer than we can see with our eyes. This picture was taken with the largest telescope in the world, the 10-meter Keck telescope on Mauna Kea in Hawaii.

The telescope took this picture the way a camcorder does, electronically. It's made of thousands of little squares, picture elements called pixels. Look at the lower right hand part of the screen, in the gray bar, and move the mouse pointer in the picture. See those numbers change? 'x' is position across the image, and 'y' the position up the image.

Those (x,y) values give the coordinates of the mouse pointer, and 'Counts' gives the brightness of that pixel.

1. How far across the screen does x go? What about y?
2. What is the total number of pixels on the screen?

Your camcorder may have as many as 400,000 pixels. Some telescope cameras have 4 million pixels.

- Zoom in and take a look at the pixels. Click on **Data Tools** in the top menu bar and select **Zoom Box**. The mouse pointer is now a cross hairs. With the mouse cross hairs at the lower left of the fireball, drag up and to the right, making a box around the fireball.
- In the **Display Controls Bar** above the picture, with the boxes that say Min, Max, Log and Zoom, go into the box that says zoom, erase the '2' that's there, and type in 20. Click on **Redraw** or press Enter/Return at the keyboard. Now you can see the pixels, the little squares the image is made of. Note the range of brightness values.
- Get back to the original *Fireball* window by clicking on some part of that window. If it is totally covered by the zoom image, move to the title bar at the top and drag the zoomed image aside.

3. How big is that fireball? Are we talking about a bonfire or something really serious? Jupiter is 11 times the diameter of Earth. How big is the fireball compared to the Earth? You can use a ruler to measure on the screen the size of the fireball and of Jupiter and set up proportions to answer this question An alternative way to measure sizes is to note the change in x or y values as you move the mouse pointer horizontally or vertically from one edge to the other of each object.
4. What is the worst thing that could happen to the Earth if a comet fragment hit it -- the very worst thing? Does a comet fragment have enough momentum to move the earth or is that like trying to knock a truck off the road with a speck of dust?

Momentum is a critical factor in assessing the impact of a crash. It equals mass times velocity. All objects in the central solar system are moving at roughly the same speed, several tens of kilometers per second. The speed of a comet relative to earth can't be much more than 60 kilometers per second or twice the speed of Earth on its journey around the sun.

Earth is over 12,000 km in diameter while the fragment was maybe 3 km -- a ratio of 4000 to 1. Let's assume the comet and Earth have about the same density, since they are both largely made of rock. Does that mean Earth has 4000 times the mass of the comet?

5. How many times more mass does Earth have than comet Fragment G?
6. How does the momentum of the comet compare with the momentum of the Earth?

The comet's momentum compared to the Earth's really is like a speck of dust compared to a truck. But, if momentum isn’t the problem with comet crashes, what is? ENERGY! The energy Fragment G released when it crashed into Jupiter was hundreds of times more than all the nuclear weapons on Earth -- all going off at once.

Kinetic energy is $\frac{1}{2}mv^2$ Squaring 60,000 m/sec is a huge number, and the mass of the comet fragment in kilograms is also huge, about 100-trillion (10^{14})kg. So $\frac{1}{2}mv^2$ is really huge. Think what an explosion of that much energy could do to Earth.

- **Open** *Jup16*, do a vertical **Flip** and Zoom 2. Adjust Min and Max to make the image look different. Try **Log** scaling.
- Add color by going to **File** and selecting **Load Color Palette**. Try *Rain.pal* as one of the best. Also try *igrey.pal*, which is an inverse grey one that gives good detail of the spots and other Jupiter features.

The spots in the lower left were made by three earlier comet fragments. They are thought to be clouds of dust and sulfur containing gases left over from the explosion of rocky comet chunks.

7. How big are the spots?
8. Could we see these spots if we took a picture of the planet tonight, or only the famous red spot (to the right and above the three spots) that was there long before the impacts? What could have happened to the spots in the turbulence of Jupiter’s fast rotating atmosphere? Where is Jupiter in the sky now anyway? Could we see it tonight?

- Use **Print** in the **File** menu to get a copy of one or both of the images.

HANDS-ON UNIVERSE™
COMPARING LUNAR CRATER HEIGHTS

OVERVIEW: In this activity you use the length of shadows as a way to compare the heights of craters' rims. You do this three ways:

Estimate the shadow lengths by eye.

Measure the shadow lengths using the cursor.

Take a Slice and measure the shadow lengths on the graph.

WHICH CRATERS ARE WE COMPARING?

The following five craters are all in the image *moon*.

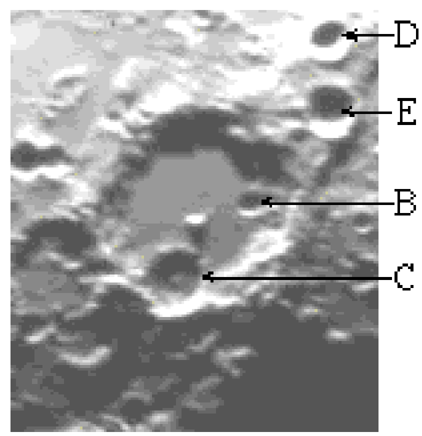
A. Albategnius, the large crater in the center of the image.

B. Crater inside Albategnius at 3 o'clock.

C. Crater inside Albategnius at 6 o'clock.

D. Crater outside Albategnius at 1 o'clock.

E. Crater outside Albategnius at 2 o'clock.



Screen Setup: A Zoom 3 or Zoom 4 image of *Moon*. Adjust the contrast using Min/Max to get the shadows as sharp as possible.

Activity I: Estimate by Eye

• Identify the five craters, A through E.

1. Without making any measurements, list the craters in order from highest rim to lowest rim. Circle ones you think are the same height.

Activity II: Measure Shadow Length Using the Cursor

The **Status Bar** shows the (x,y) coordinates for the cursor and the brightness in Counts at that pixel. Use the down-arrow key to move the cursor down over a shadow. Moving vertically simplifies the math since the distance in pixels will only be the change in the y-coordinate. Deciding on the beginning and end points of a shadow is a judgment call – what brightness level is the edge between shadow and lit? Using **Zoom Box** on the crater you are measuring can help. If the magnified image is larger than the window, you may need to scroll in order to see the shadow you are working with.

2. What direction is the sun coming from?

3. Record your cursor data for each crater.

Activity III: Use the Slice Graph To Measure Shadow Lengths

- Use **Slice** from the **Data Tools** menu to draw vertical slices with the same x-coordinates you used in Activity II when measuring the shadows with the cursor. Start each slice above the shadow and extend below. Arrange your screen so you can see both the *Slice* and *Moon* windows.
- See *An Introduction to Image Processing*, Activity III, for how to enlarge the Slice window, display graph values, and add tick marks and a grid.

4. Record your Slice data for each crater.

5. Collect all your data.

Crater shadow lengths measured using the cursor:

Crater shadow lengths measured using the *Slice* graph:

6. Order the crater rims from highest rim to lowest rim and compare these with your earlier estimate by eye.

7. If these orderings are not the same, point out the differences and speculate on why they occurred.

Were your estimates by eye off because of the way the image appeared?

Did your measurements differ because of the nature of the tool - cursor readings versus *Slice* graph?

Date: ______________ Name: ________________________

Answer Sheet

Comparing Lunar Crater Heights Unit

Activity I: Estimate by Eye

1. My eye estimate of the order of the crater rims from highest rim to lowest rim. Circled ones are ones I think are the same height.

1. __ 2. __ 3. __ 4. __ 5. __

Activity II: Measure Shadow Length Using the Cursor

2. What direction is the sun coming from?

3. Cursor data for each crater.

Which crater: __
Start of shadow: (x,y) = ____ Brightness: ____
End of shadow: (x,y) = ____ Brightness: ____
Length of shadow: _____

Which crater: __
Start of shadow: (x,y) = ____ Brightness: ____
End of shadow: (x,y) = ____ Brightness: ____
Length of shadow: _____

Which crater: __
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Length of shadow: _____

Which crater: __
Start of shadow: (x,y) = ____ Brightness: ____
End of shadow: (x,y) = ____ Brightness: ____
Length of shadow: _____

Activity III: Use the Slice Graph To Measure Shadow Lengths

4. Slice data for each crater.

Which crater: __

Start of shadow: (x,y) = ____ Brightness: ____

End of shadow: (x,y) = ____ Brightness: ____

Length of shadow: _____

Which crater: __

Start of shadow: (x,y) = ____ Brightness: ____

End of shadow: (x,y) = ____ Brightness: ____

Length of shadow: _____

Which crater: __

Start of shadow: (x,y) = ____ Brightness: ____

End of shadow: (x,y) = ____ Brightness: ____

Length of shadow: _____

Which crater: __

Start of shadow: (x,y) = ____ Brightness: ____

End of shadow: (x,y = ____ Brightness: ____

Length of shadow: _____

Which crater: __

Start of shadow: (x,y = ____ Brightness: ____

End of shadow: (x,y = ____ Brightness: ____

Length of shadow: _____

5. Collected data.

Crater shadow lengths measured using the cursor:

A: _____ B: _____ C: _____ D: _____ E: _____

Crater shadow lengths measured using the *Slice* graph:

A: _____ B: _____ C: _____ D: _____ E: _____

6. Order of crater rims from highest rim to lowest rim.

order by cursor measurement: 1. _ 2. _ 3. _ 4. _ 5. _

order by *Slice* measurement: 1. _ 2. _ 3. _ 4. _ 5. _

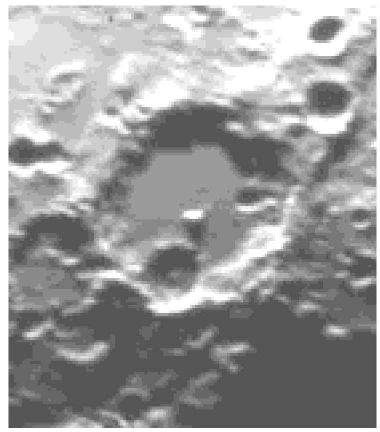
and my original estimate by eye.
order by estimate: 1. _ 2. _ 3. _ 4. _ 5. _

7. For differences in the three ways of ordering, here is why I think they occurred.

HANDS-ON UNIVERSE™

INVESTIGATING MOON CRATERS UNIT

The Challenge: Use the image processing tools to try to answer questions about the topography of the moon, such as some of the questions listed below. They are not all easy ones, and some may require more than a full class period to answer.



When you have some answers, write them down. Your write-up for each question you choose to work on should include:

- The Question.
- Your Answer.
- A Description of how you used the image processing software to help you find your answer.
- A Comparison of your answers with classmates' answers. If you disagree, include in your write-up any resolutions of these differences, or defend your version.

Examples of Questions About the Craters in the Moon Image:

- How many craters are there?
- Using the shadows as an indication of height, which crater has the highest walls around it? How high?
- What is the range of crater sizes?
- Some of the craters have a central peak in the middle of the basin. How does the height of the peak compare with the height of the outer walls?
- What are the relative ages of the craters from most recent to oldest?
- What about other formations than just craters? Where are some? Speculate on what they might be.

(After studying the moon image, add at least one of your own questions here)

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Suggestions for Finding Answers:

Screen Setup: Two *Moon* images at Zoom 3 or 4 with the contrast adjusted to bring out detail. In this way you can do image processing in one window and compare with the other to see what difference it made.

To get a magnified view use **Zoom Box** in **Data Tools** and drag the cursor to form a box around a portion of the image.

To do graphical analysis use **Slice** in **Data Tools** to get a graph of the brightness in Counts along the Slice. This gives you a way to see changes in brightness and to find Counts and pixel distances.

To deal with multiple windows use **Tile** or **Cascade** in the **Window** menu to simplify working with multiple windows on the screen. In the **Window** menu you can click on the one you want to work with to bring it up front on the screen as the active window.

If scroll bars appear on a window, it can mean Tiling or Cascading has reduced the window size. Click on the top right corner of the image screen to bring the window back to its original size. The window needs to be away from the bottom and right edges of the screen in order to have room to expand back to its original size.

HANDS-ON UNIVERSE™

TRACKING JUPITER'S MOONS UNIT

Introduction.

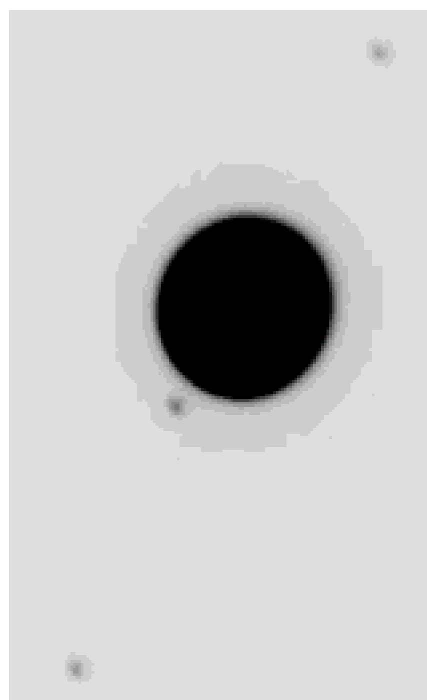
Galileo discovered the four largest moons of Jupiter in 1610, and they are often referred to as the Galilean Moons. He was using a simple telescope and a keen mind. It is a testimony to his observational prowess that out of all the stars and bright objects he could see in the sky, he noticed that Jupiter and these four dimmer lights, which he assumed were stars, were stretched out along a straight line. When he looked again he saw that the positions had changed from one night to the next, which is not what stars do. After repeated observations he determined that they were moons orbiting around Jupiter.

The *Jup5* to *Jup10* images of Jupiter and its Galilean moons were taken at one-hour intervals on April, 1992.

Activity I: Find the Moons

Screen Setup: *Jup5* and *Jup6* with the contrast adjusted using Min/Max so the moons are visible.

To help keep track of the moons, refer to them as # 1, 2, 3, & 4, starting from the bottom of the window. To see all four of Jupiter's moons you may need to scroll the image within the window by clicking inside the scroll bar, or pressing on an arrow button to scroll by smaller steps, or dragging the scroll bar to make larger scrolls. (Click, Press, or Drag)



1. Record your image settings.

Which Direction is Each Moon Moving?

• You can tell by eye which direction the two moons shown closest to Jupiter are moving. A way to answer this question for the other two moons is to compare position coordinates.

• Use **Find** in the **Data Tools** menu with the default setting. Do this for both images.

2. Make a sketch of the *Jup5* image, or use the **Print** option in the **File** menu. Based on your information from *Jup5* and *Jup6* in the *Results* window, draw an arrow at each moon showing whether it appears to be moving closer to or further away from Jupiter..

Activity II: Making a Double Exposure.

Adding two images together to make a double exposure is another way to compare the positions of the moons in two images taken an hour apart.

Screen Setup: *Jup5* and *Jup6* with the contrast adjusted so the moons are visible.

• Starting with *Jup5* as the active window, use **Add** in the **Manipulation** menu. Click on **Displayed image** and scroll down to select *Jup6* for what to add. Click on **Display result in new window.**

• If you want to save this double image, select **Save As** from the **File** menu and enter a new file name including your initials, such as "Jup56jd".

• Use **Find** to get the brightness Counts for the Sky and the moons in all three images.

3. Adding the two images made the Sky roughly twice the value in either single image. The moons, however, are not twice as bright. How come?

• Which image did each moon come from? Compare moon coordinates in the double image with moon coordinates in one of the single images.

4. Make a sketch, or **Print** out a copy, of the double image and draw in arrows from the *Jup5* position to the *Jup6* position of each moon.

Activity III: What Happens to the Moons During Six Hours?

For this task you need to collect data on the positions of each of the moons in each of the six Jupiter images, *Jup5, Jup6, Jup7, Jup8, jup9* and *Jup10*, taken at one hour intervals.

5. For each image check the **Image Info** (under **Data Tools**) and record the date and time that the image was taken. Date is day/month/year and time is Universal Time, UT. Make a quick sketch of the image. Universal Time is the time in Greenwich England.

In order to see how each moon moves during the time sequence represented by the six images, combine all six shifted images into one composite image. This may be done in several ways: by adding all of them together at one time; by adding them together one at a time and checking after each addition; by subtracting some and adding some. You may think of some other ways.

Try whatever you like. Once again keep a careful record of all that you do, including the names of the files you create and how you create them. Remember, your goal here is to create an image or images that will allow you to see as clearly as possible how these Moons are moving.

- Select **Cascade** or **Tile** in the **Window** menu to help manage multiple windows. Select the window you want to work with from the **Window** menu to bring it to the front as the active window. If the window has been reduced, maximize by clicking in the top right box of the screen. In order to maximize the window needs to be away from the bottom and right edges of the screen.

Here are ways to collect moon position data; you may think of more. Decide which works best for you.

Use the cursor to get the coordinates for the positions of each moon.

Use **Slice** in the **Data Tools** menu to get the number of pixels between each moon and the center of Jupiter or between moon positions. (It helps to make the *Slice* window larger - on a PC by dragging the borders, on a Mac by dragging the bottom right corner.) Drag the cursor on the *Slice* graph to display values for distance along the Slice in pixels and brightness in Counts. Corresponding pixel (x,y) coordinates and Counts are shown in the **Status Bar -** be sure you understand the differences between the (x,y) values for the Image window and the values shown on the Slice graph.

Use **Find** to get cursor coordinates for all six positions of each moon and use the Pythagorean Theorem to compute distances and speed. A hand calculator helps here.

6. Make a sketch (or get a printout, if possible) of your composite image. If you sketch it, please take enough time so that it's clear to someone else who looks at it. Share your results with other groups around you and see what approach they used that might be different from yours. This is particularly valuable as you begin to answer the questions below.

7. Identify on your sketch the orbits each moon is traveling in by putting the number of the moon at its initial position in *Jup5* and in its last position in *Jup10.*

8. Which moon(s) appear to be traveling the fastest? slowest? Does this depend on the portion of the orbit you are examining? Explain your reasoning.

9. Record the direction and speed of each moon. Your units of speed will be either pixels/hr or mm/hr, depending on your method of collecting the data.

The moons all move at roughly constant speeds around Jupiter in almost circular orbits. Despite this fact, in analyzing your data you should have found that the speed seemed to change. This is particularly true for one of the moons.

10. How do you explain this apparent paradox?

11. Draw a top view of Jupiter and each moon in its six successive positions.

Activity IV: Interpreting Your Data

- The four moons Galileo discovered in 1610 are named Io, Europa, Ganymede, and Callisto. Here is the period and orbit radius for each moon. The period is the time for one complete revolution.

Moon	Period (days)	Orbit Radius (km)
Io	1.8	421,600
Europa	3.5	670,900
Ganymede	7.2	1,070,000
Callisto	16.7	1,883,000

One more piece of information: the further the orbit from Jupiter, the slower the speed of the moon. This is because Jupiter's gravity weakens with distance.

12. Who Is Io? For each moon, see if you can match the name with its number. Use a process of elimination, crossing out numbers that are not candidates.

13. Explain how you decided on the name for each moon.

Date: ______________ Name: _______________________

Answer Sheet

Tracking Jupiter's Moons Unit

Activity I:

1. My image settings:

Color Palette: _______________

Min: _____ Max: _____ Log scaling (y/n): ___

2. Which way each moon is moving (see my attached sketch or printout).

Activity II:

3. Why I think the moons are not twice as bright.

4. The double exposure. (See attached paper.)

Activity III:

5. Image Info: Date and Universal Time

	Date	Universal Time
Jup5	_________	_________
Jup6	_________	_________
Jup7	_________	_________
Jup8	_________	_________
Jup9	_________	_________
Jup10	_________	_________

6 & 7. See attached sketch.

8. Fastest moon #: ____

Slowest moon #: ____

My reasoning in selecting these answers:

9. Direction and speed of each moon.

Moon #1:

Direction of motion relative to Jupiter:

Speed relative to Jupiter:

Moon #2:

Direction of motion relative to Jupiter:
1

Speed relative to Jupiter:

Moon #3:

Direction of motion relative to Jupiter:

Speed relative to Jupiter:

Moon #4:

Direction of motion relative to Jupiter:

Speed relative to Jupiter:

10. How I explain the apparent paradox.

11. A top view. (See attached drawing.)

Activity IV:

12. Who Is Io? Identity each of the four moons.

Io:	#1?	#2?	#3?	#4?
Europa:	#1?	#2?	#3?	#4?
Ganymede:	#1?	#2?	#3?	#4?
Callisto:	#1?	#2?	#3?	#4?

13. How I decided on the name for each moon.

HANDS-ON UNIVERSE™
SUPPLEMENTARY ACTIVITY 3
SIMULATING ORBITS

This activity is designed to acquaint you with the properties of the orbital motion of the moons around Jupiter when looking at the orbits from the edge. For this activity you'll need lined paper, graph paper, a compass and a protractor.

1. Before starting the written activities below, spend some time observing the ball that is attached to the edge of the rotating turntable. View it from the top and from the side. Particularly from the side, try to watch only the motion of the ball, not the turntable. What differences do you notice in the two views?

2. On the lined paper (with the lines running vertically) draw a circle that fills the page.

3. Draw a single horizontal line from one side of the circle, through the center, to the other side. Label the intersection of the line and the left side of the circle with a "0".

4. From point "0" use a protractor to mark points all the way around the circle at every 10 degrees.

5. Mark these points with consecutive numbers up to 36. These points will represent consecutive positions of a moon as it orbits Jupiter.

6. Draw light vertical lines (aligned with the lines on the paper) from each point you have marked on the circle down to the horizontal line. Do this for half of the circle. Make small dots where these lines intersect the horizontal line and label them 0 to 12. These dots represent the 'apparent position' of the moon as we see it from the side.

7. Assume that the time between each dot in your diagram represents 1 hour. The distance scale will be 1 cm = 100,000 km (10^5 km.) Use the information from the diagram to do the following:

a) Calculate the apparent average velocity of the moon between the position 0 and 1, 1 and 2, 2 and 3, etc. for half an orbit. Please find these numbers in units of km/hr and record them in a table. You may save yourself some time by finding a pattern to the calculations.

b) Construct a graph of apparent average velocity vs. position number. (*Remember that you have calculated an average velocity that is the value in the middle of the interval.) Include a few values of the velocity after the "turnaround point" on the right side of your circle. Consider velocities moving toward the right positive; those toward the left negative.

8. Answer the following questions based on your work above:

a) Where does it appear that the moon is traveling the fastest? the slowest?

b) Where is your velocity calculation close to the actual (tangential) velocity?

9. Using the circle diagram prepared in #6, construct a second graph of apparent distance from Jupiter vs. position number. Distances to the left of Jupiter are negative; those to the right are positive. This should be drawn for one full period of the orbit.

10. Label the following positions on the two graphs you have created:

(a) turnaround point, (b) occultation region, (c) transit region.

11. Given that you only have the side view of the moon's orbit, which position(s) would you need to use to determine the actual radius of the moon's orbit? Explain.

12. What minimum portion of the graph in #9 could you use to determine the period of the moon's orbit? Suppose you had less than this, what might you do to make this determination?

13. Using a scientific calculator, make a table of sin(x) vs. x for values of x from 0 to 360 degrees. Graph this information. (If you have a graphing calculator use it, and then sketch the graph you see.) How does the shape of this graph compare to those you've drawn above?

14. Look at the Moons of Jupiter orbits published in *Sky and Telescope* or *Astronomy* magazine. What similarities do you see between these and your work above?

HANDS-ON UNIVERSE™
SUPPLEMENTARY ACTIVITY 4
USING LARGE AND SMALL NUMBERS

How High Can You Count?

• Start counting quietly and time yourself.

1. How long does it take you to count to 10?

2. How long does it take you to count to 100?

3. At the same rate, how long would it take you to count to 1000?

4. How high could you count in a day?

5. How high could you count in your lifetime?

Powers of Ten

• Go around the room and each person take a turn.
The first person thinks of an object the size of a millimeter (1 x 10^{-3} m).
The second person thinks of an object the size of a centimeter (1 x 10^{-2} m).
The next person thinks of an object the size of 10 centimeters (1 x 10^{-1} m)….

•. Keep multiplying by ten and see how high you can go, figuring out the power of ten and thinking of objects for each size. Keep a list of the objects.

6. How tall are you in centimeters? in meters?

7. How far are you from your home in meters? in kilometers?

8. What is the farthest you have ever traveled in meters?

9. What is the circumference of the Earth in meters?

• Starting with one centimeter, this time keep dividing by ten, thinking of objects each time.

10. Referring to your lists of objects, what are two objects whose ratio of sizes is:

1000 to 1?

1 x 10^{5} to 1?

1 x 10^{-6} to 1?

HANDS-ON UNIVERSE™
SUPPLEMENTARY ACTIVITY 5
EXPLANATION OF A LIGHT YEAR

1. Have one person run down the hallway or outside on the sidewalk at a constant rate.

How far does s/he run in one second?

This is a unit of measurement for distance that could be called a "*Jane Doe*- second" (if Jane was the person running).

2. Using this new unit of distance, estimate:

A. The length of the hallway.

B. The height of the school building.

3. Consider a car that is traveling at 30 miles per hour. Convert to meters and calculate the length of a "car-minute". Use the unit of "car-minutes" to estimate the distance from school to your home.

4. For many of the following set of questions you will need to go to a reference to look up distances from Earth to other astronomical objects.

In a vacuum (which we will consider a good approximation for outer space) light travels at 300,000 kilometers per second (186,000 miles per second).

A. How many times faster is this than a car obeying the speed limit on a highway?

B. How many times faster is it than an average airplane in flight (500 miles per hour)?

C. Calculate how far light travels in one second (in meters).

D. How many light-seconds is the Earth-Moon distance?

E. Calculate how far light travels in one minute (in meters).

F. How long has the light we receive from the Sun been traveling through space?

G. Calculate how far light travels in one year (in meters). **This is a light year.**

H. If a message, traveling at the speed of light, were sent out into space on the day you were born, how far would it have gone by now?

I. Can you find an astronomical object that could be receiving your birthday message this year? (If not, find an object that could receive your message soon and specify how soon.)

J. How long does it take for us to receive light traveling from the farthest planet in our solar system?

K. How long does it take light to travel from one side of our galaxy to the other?

L. Find an astronomical object for which the light we see now has been traveling to us since humankind began (about 2 million years ago).

M. Betelgeuse is a star in the constellation Orion that is close to dying a violent death called a supernova. If the star exploded now would you be likely to see the bright flash in your lifetime? Explain your answer.

N. Suppose you could determine the distance to the farthest object in the universe from which we are receiving light. Explain how this could give you an estimate of the age of the universe.

Date: _______________ Name ______________________

Answer Sheet

Supplementary Activity 5

Light Years

1. How far s/he ran in one second: ______

2. Using this new unit of distance, my estimates are:

A. The length of the hallway. ______

B. The height of the school building. ______

3. Distance from school to my home in car minutes ______

4. Given light travels at 300,000 kilometers per second (186,000 miles per second).

A. How many times faster this is than a car obeying the highway speed limit: ______

B. How many times faster this is than an average airplane (500 miles per hour): ______

C. Distance light travels in one second (in meters): ______

D. The Earth-Moon distance in light-seconds: ______

E. Distance light travels in one minute (in meters): ______

F. Time for light from the Sun to reach us: ______

G. How far light travels in one year (in meters)? **This is a light year.** ______

H. The distance traveled by a message, traveling at the speed of light and sent out in to space on the day I was born: ____

I. An astronomical object receiving my birthday message and when:

____

J. Time to receive light from the farthest planet in our solar system: _______________

K. Time for light to travel from one side of our galaxy to the other: _______________

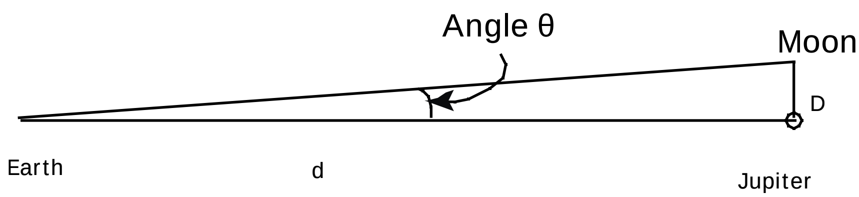
L. An astronomical object, just now seen, whose light has been traveling to us since humankind began (about 2 million years ago). ______________________________

M. Why I would or would not see Betelgeuse in my lifetime if it became a supernova now:

N. How to estimate the age of the universe if we knew the distance to the farthest visible object in the universe:

HANDS-ON UNIVERSE™
SUPPLEMENTARY ACTIVITY 6
USING ANGLES TO MEASURE SIZES

Generally, astronomical objects are so far away, Earthlings will never have the chance to go to them and take measurements such as size, distance, mass, and temperature. Therefore, methods must be developed to infer these characteristics from measurements we can make from Earth. The apparent size in the sky of an object, or a feature such as the radius of a moon's orbit, depends on the actual size of the object, D, and its distance away from us, d. (See Diagram below). Imagine lines reaching from the point at which you are observing to each side of the object. The angle between these two lines, θ, is the angle subtended by the object.



When θ is small (much less than 1 radian) and expressed in radians (2π radians = 360°), the following formula can be used:

$$\mathbf{D = d \times \theta}$$

This is commonly called the **Small Angle Approximation**, because it is valid only when
θ is much less than 1 radians. (See the *Measuring Size With Images Discussion Sheet* for more on the Small Angle Approximation.)

Activity I: Measurements in Degrees

1. Look up the necessary information and use the Small Angle Approximation to find:
  A. The average subtended angle of the Moon as viewed from Earth.
  B. The average subtended angle of the Sun as viewed from Earth.

The fact that the two subtended angles are so close is purely a coincidence of nature. If the subtended angle of the Moon were smaller than that of the Sun, we would not observe total eclipses of the Sun. If the subtended angle of the Moon were larger, we would observe total eclipses more often; however, the corona of the Sun would not be visible during the eclipse. The corona appears as a beautiful, bright halo around the Sun and provides a unique opportunity for astronomers to study the outer layer of the Sun during an eclipse.

You may have noticed that a full Moon appears to change size during the night. Try to remember the last time you saw a full Moon while it was rising and also when it was nearly overhead in the sky.

2. At which point did the Moon appear largest?
3. When did the Moon appear smallest?

You can approximate measurements of the Moon, or any relatively large object in the sky, by using your own body. Stand up and hold your arm straight out in front of you. Make sure it is parallel to the ground. Close your fists and stack one fist on top of another (with straight arms) until your arm is now pointing straight up overhead. Take care to keep the lower hand stationary when you stack the other hand on top of it.

4. How many fists does it take to form the 90° angle between parallel to the ground and straight overhead (it will probably be between 7 and 10)?
5. Divide 90° by the number of fists to get the number of degrees that your fist subtends (e.g., if it took 9 fists to get overhead, then each fist subtends 10°).

You now have a measuring stick to describe angles in the sky. You can get a more accurate ruler by using fingers. Find an object in the distance that subtends the same angle as your fist. Now hold your hand out flat and count how many finger-widths it takes to cover the object (it will probably be 4 or 5 depending on how you form a fist).

6. Divide the number of degrees subtended by your fist by the number of fingers that fit into that angle. This is the number of degrees subtended by one finger.

Another method is to use your thumb, held out as if you were giving a thumbs-up sign, and determine how many thumbs cover the angle subtended by your fist. The advantage of this technique is that your thumbnail provides a curved edge, which probably subtends about the same angle as the Sun or Moon.

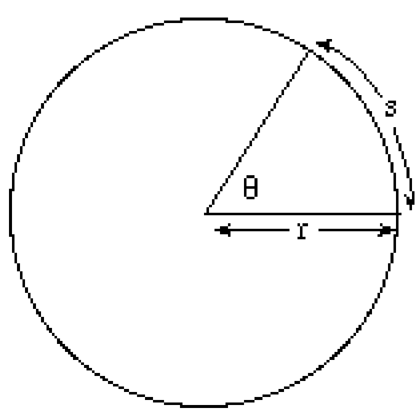
At full Moon, use your thumb or finger as a measuring stick to estimate the subtended angle of the Moon when it is rising and also when it is overhead. You may be surprised!

A variety of units are used to describe angles. Which one you use usually depends on how you are going to use the measurement or the approximate size of the angle you are measuring. You are probably most familiar with degrees since this is the unit most people use to describe angles. When you are using the Small Angle Approximation, however, you must use the unit radians to describe an angle. Finally, astronomers often use the unit arc seconds, often abbreviated with a double quote symbol ("), to describe angles because the angles commonly used in astronomy are very, very small. The next two activities will familiarize you with these three units for angles.

There are 360° in a circle.
There are 2π radians in a circle.
1 degree = 0.017 radian = 3600"
1 radian = 57.3° = 206,265"
1" = 0.00028° = 0.0000048 radians (4.8 x 10^{-6} radians)

Activity II: Measuring Angles with Radians

A radian is defined as the angle, θ, when the arc length, s, of the inscribed angle is equal to the radius r.



- Use a compass to draw several circles of various radii, or alternatively, trace the circumference of objects with a circular base (such as tin cans of varying size).

7. Use a ruler to measure the radius of each circle. Label this on each drawing.

- Devise a technique to measure the arc length, s, between any two points on the circle. Using this technique, measure an arc length on each circle that is equal to the radius of that circle. Mark each end of the arc length and label the arc "s".
- Draw lines from the center of the circle to each endpoint of s (so it resembles the figure).

8. Use a protractor to measure the angle formed between the two lines in degrees. Convert your measurement to radians. Label each circle with these values.

Activity III: Measuring Very Small Angles

With at least three people, take a long piece of string and a protractor into a long hallway. Have one person stand at one end of the hall with the two ends of the string while another person takes the midpoint of the string toward the other end of the hallway so that it forms the vertex of an angle.

9. The third person measures the angle at the vertex and the distance between the endpoints.

10. Use the Small Angle Approximation to calculate the angle formed by your string and compare it to your measurement. Remember that the angle determined using the Small Angle Approximation is in radians.

11. Create the smallest angle with the string that you can measure accurately. Express this angle in degrees, in radians, and in arc seconds.

12. Can you form an angle that is one radian? one degree? one arc second? Explain your answers.

HANDS-ON UNIVERSE™
SUPPLEMENTARY ACTIVITY 7
MEASURING YOUR COMPUTER SCREEN

Work in a small group to measure the subtended angle of your computer screen when you are sitting in a chair at least one meter away from it. You will measure this angle in two different ways and compare the answers.

First Technique: One person sits in the chair as the observer, while others hold strings leading from a point between the observer's eyes to each edge of the computer screen. Make sure the strings are pulled tightly so they form straight lines. Another person uses a protractor to measure the angle formed between the two strings.

Second Technique: Measure the width of the screen and the distance between the observer and the screen. Use the Small Angle Approximation,

$$D = d \times \theta$$

to calculate the subtended angle. (See the *Measuring Size with Images Discussion Sheet* for an explanation of the Small Angle Approximation.)

1. What is the angle subtended by the screen (in radians) using each technique?

2. Convert your measurements to degrees.

3. Would the second technique be valid if the observer were one foot away from the screen? Why or why not?

HANDS-ON UNIVERSE™
DISCUSSION SHEET
MEASURING SIZE WITH IMAGES

A CCD image is made up of little boxes called pixels. Each pixel on the image represents a very small portion of the sky and is colored or shaded to represent the amount of light received through the telescope from that part of the sky. If you consider the entire sky to be a sphere surrounding us, you can imagine the field of view of the telescope as covering a small patch on that sphere and each pixel of a CCD covering a much smaller patch.

Plate Scale of a CCD Image

The plate scale of an image is the measure of the angle represented in the sky by each pixel on the image. For instance, the plate scale for the 30-inch telescope at Leuschner Observatory, where most of the unit images were taken, was originally 0.99 arcsecs/pixel and then when they got a new CCD it was 0.67 arcsecs/pixel. This means that each pixel on the image represents 0.67 arc seconds in the sky.

You may ask, "What is an arc second?" Generally, the angle covered by a star, or even a galaxy, is much smaller than a degree, so astronomers have developed their own units to describe very small angles. When dealing with astronomical images, arc seconds are usually used and occasionally arc minutes.

1° is 1/360th of a circle
1° = 60 arc minutes (written 60')
1' = 60 arc seconds (written 60")
therefore 1" = 1/3600°, which is a VERY small angle.

You can measure the plate scale in degrees, radians, arc seconds or any other units to describe angles, but most of the time a plate scale will be provided in arc seconds per pixel.

Using Angles to Measure Size: The Small Angle Approximation

The angle covered by an object depends on both its actual size and the distance to the object. Since most astronomical objects cover very small angles, the Small Angle Approximation can be used to show the relationship between angle, distance and size of objects. Here is an explanation of the Small Angle Approximation.

Consider a circle having a radius equal to the distance, d, between the observer and the object. In Diagram 1, D is the width of the object, θ is the angle subtended by the object, and s is the arc length subtended by the object.

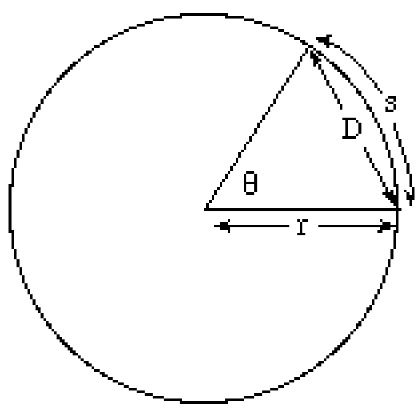


Diagram 1: A radian

A radian is a measure of the angle formed when the arc length equals the radius; i.e., s = r. For a full circle, the arc length subtended is the circumference ($s = c = 2\pi r$), which means the angle inscribed in a full circle is 2π radians.

$$360° = 2\pi \text{ radians} \quad \text{or} \quad 1° = 2\pi/360 \text{ radians} = 0.017 \text{ radians}$$
$$\text{and } 1 \text{ radian} = 57.3° = 206{,}265''$$

The relationship between s and r can be written more generally in terms of any angle, θ, that is measured in radians:

$$s = r \times \theta \qquad \text{when } \theta \text{ is measured in radians.}$$

As θ becomes smaller, the arc length, s, has less curvature and can therefore be approximated by a straight line, D (see Diagram 1). When θ is much less than 1 radian or so, the lengths of D and s become almost equal, which gives us the Small Angle Approximation:

$$\mathbf{D = d \times} \theta$$

The letter d, for distance away, is now used instead of r.

In summary, when the angle, θ, covered by the object is much less than 1 radian, then the size of the angle in radians is approximately equal to the ratio of the diameter or width of the object to its distance away (see Diagram 2).

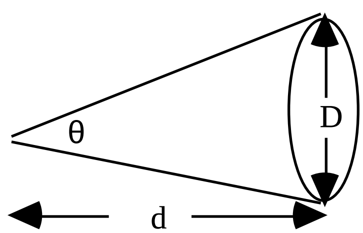


Diagram 2

You need to be very careful to keep your units of measurement straight when working with plate scales and the Small Angle Approximation. In order to measure sizes of the objects on CCD images, there are four steps:

1) Measure the number of pixels covered by the object
2) Use the plate scale for the image to calculate the actual angle in the sky covered by the object.
3) Convert this to radians.
4) Use the Small Angle Approximation to calculate the size of the object given its angle in radians and distance away.

HANDS-ON UNIVERSE™

MEASURING SIZE WITH IMAGES UNIT

Activity I: What is a Pixel?

• **Open** any HOU image using any color palette you choose. Use **Zoom** to enlarge the image until it appears to be made up of small squares. These squares are called pixels, which is short for picture elements.

1. Within each square, does the color or shading vary?

2. If you **Zoom** the image even more, does that color within each pixel change?

Activity II: Measuring Plate Scale of an Image

Screen Setup: *Eclipse1*

The image is of a solar eclipse taken in 1991 when the Moon passed directly in front of the Sun. The angle covered in the sky, called the subtended angle, by both the Sun and the Moon is 1/2°.

3. Use cursor readings in the **Status Bar** or use **Slice** under **Data Tools** to measure the number of pixels across the width of the Moon in the image.

4. Calculate the ratio of the angle covered by the Moon in the sky when looking at it with your naked eyes to the number of pixels covered by the Moon in the image. This is the plate scale of this image in units of degrees/pixel.

5. Plate scales of CCD images are commonly expressed in arcsecs/pixel. Use the conversion factor, 1 degree = 3600" (A double quotation mark, ", is the common symbol for arc seconds) to calculate the plate scale of the image of the eclipse in arcsecs/pixel.

Activity III: Measuring Size on a CCD Image

Screen Setup: *Rori*

The image is of Rori, an HOU student, sitting at her computer. You can use the **Rotate** or **Flip** option under **Manipulation** to make the image right side up. This is a large image so rotating it may take a few minutes.

6. Use cursor readings in the **Status Bar** to determine the number of pixels in the width of the screen on Rori's computer.

7. Given the following data:

The width of Rori's screen = 11 inches

The distance of the camera from the screen = 6 feet 9 inches

use the Small Angle Approximation to calculate the angle covered by Rori's screen as observed by the camera. (See the *Measuring Size with Images Discussion Sheet* for an explanation of the Small Angle Approximation.). This angle is rather large for the Small Angle Approximation but will suffice for this activity.

8. Calculate the plate scale of the *Rori* image in arcsecs/pixel. (1 radian = 206,265")

Activity IV: Measuring the Size of Astronomical Objects

In this activity, you may first need to do some image processing (using the Min/Max adjustment and Log scaling) to make sure you are measuring the entire width of the object.

Screen Setup: *Moon*

Object: any one of the craters

Plate Scale: 0.99"/pixel

Distance to the Moon: 3.84×10^8 m

9. Find the width of the crater in pixels.

10. Use the plate scale to calculate the angle (in arc seconds) subtended by the crater.

11. Use the Small Angle Approximation and the distance provided to get the actual size of the crater. Remember to convert the angle to radians.

12. Could a house fit inside this crater?

Screen Setup: *Jup1*

Object: Jupiter

Plate scale = 0.67"/pixel

Average distance to Jupiter = 7.8×10^{11} m

13. Find the width of Jupiter in pixels.

14. Calculate the angle subtended by Jupiter.

15. Calculate the diameter of Jupiter.

16. How many times bigger is Jupiter than the moon crater you measured?

Screen Setup: *Eclipse1*

Object: Sun during a solar eclipse

Plate scale = 3.0"/pixel

Average distance to the sun = 1.5×10^{11} m

17. Find the width of the Sun in pixels.

18. Calculate the angle subtended by the Sun.

19. Calculate the diameter of the Sun.

20. How does this compare with the size of Jupiter?

Screen Setup: *Crab*

Object: the Crab Nebula (a supernova remnant)

Plate scale = 0.99"/pixel

Distance to the Crab Nebula = 6000 light years (ly)

1 ly = 9.5×10^{15} m

21. Convert the distance to the Crab Nebula from light years to meters.

22. Find the width of the Crab Nebula in pixels.

23. Calculate the angle subtended by the Crab Nebula.

24. Calculate the diameter of the Crab Nebula.

25. How does the width of the Crab Nebula compare to the Earth-Sun distance?

Object: M51, a spiral galaxy

Plate scale = 0.99”/pixel

Distance to the galaxy M51 = 30 million ly

1 ly = 9.5×10^{15} m

26. How wide are the spiral arms in light years?

27. How wide is the entire galaxy?

28. How does this compare to the width of the Crab Nebula?

Final Challenge:

Determine the field of view for one of the images used in Activity IV. The field of view is the angle covered by the entire image.

Date: ______________ Name: ________________________

Answer Sheet

Measuring Size with Images Unit

Activity I: What is a Pixel?

1. Variation of color or shading within a pixel: ________________

2. The effect of **Zoom** on the variation: ________________

Activity II: Measuring Plate Scale of an Image

3. Number of pixels covered by the moon: ________________

4. Plate scale of *eclipse1* image in degrees/pixel: ________________

5. Plate scale of the *eclipse1* image in arcsecs/pixel: ________________

Activity III: Measuring Size on a CCD Image

6. Width of Rori's screen in pixels: ________________

7. Angle covered by Rori's screen: ________________

8. Plate scale of the *Rori* image: ________________

Activity IV: Measuring the Size of Astronomical Objects

9. Width of moon crater in pixels: ________________

10. Angle covered by crater in arc seconds: ________________

11. Actual size of crater in meters: ________________

12. Why a house could or could not fit in this crater: ________________

13. Width of Jupiter in pixels: ________________

14. Angle subtended by Jupiter in arc seconds: ________________

15. Diameter of Jupiter in meters: ________________

16. The size of Jupiter compared to the moon crater: ________________

17. Width of sun in pixels: ____________________

18. Angle subtended by Sun in arc seconds: ____________________

19. Diameter of Sun in meters: ____________________

20. The size of the Sun compared to Jupiter: ____________________

21. Distance to the Crab Nebula in meters: ____________________

22. Width of Crab nebula in pixels: ____________________

23. Angle covered by the Crab Nebula in arc seconds: ____________________

24. Diameter of the Crab Nebula in meters: ____________________

25. Width of the Crab Nebula compared to the Earth-Sun distance: ____________________

26. Width of the spiral arms of M51 in light years: ____________________

27. Width of M51: ____________________

28. Width of M51 compared to the Crab Nebula: ____________________

Final Challenge:

Image chosen: ____________________

Field of view of entire image: ____________________

HANDS ON UNIVERSE™

MEASURING THE SIZE OF MOON FEATURES UNIT

Overview

In this unit you will use simple ratios to determine the estimated size of some of the features in the image, *moon,* such as the diameter of a crater, the height of a mountain, or the height of a crater wall.

The *moon* image was taken on the night of October 17th, 1988 by the 20 inch telescope at Leuschner Observatory in the Lafayette Hills just 5 miles east of Berkeley, California. The *plate scale* for the Charge Coupled Device, the CCD (i.e., the electronic detector attached to the telescope), is 0.99"/pixel. This means that each pixel in the image corresponds to an angle of 0.99 arc seconds in the sky, or 0.99".

Definitions:

The angle subtended refers to the angle between the sight lines to opposite edges of the object.

1 degree = 60 arc minutes: $1^{o} = 60'$

1 minute = 60 arc seconds (arcsecs): 1' = 60"

1 pixel = 0.99 arcsecs (CCD plate scale for the 20" Leuschner detector)

Moon angle subtended = 1/2 degree or 1800 arcsecs

1 kilometer (km) = 5/8 miles

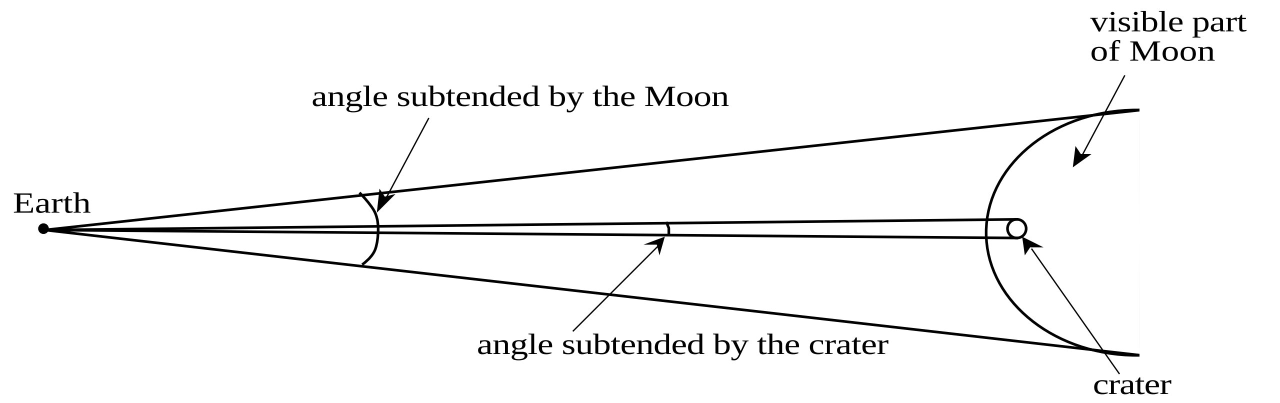
Moon diameter = 3,476 km = 2,172 miles

Activity I: Find the Size of the Largest Crater

1. Use Image Processor tools to find the diameter of the largest crater in pixels.

2. Convert pixels to arc seconds using the CCD plate scale.

When we look at objects that are very far away or very small, we can use what is called the Small Angle Approximation together with a relationship between arc length, distance and subtended angle to form ratios that help us find actual sizes of objects. For this part we will consider looking at a Moon crater.



Enlarged image of the Moon, showing imaginary lines of sight from the Earth to the Moon. The diagram is not to scale and the point of view has been shifted to help show the angles.

Diameters of features on the moon, such as craters or even the whole Moon, are proportional to their subtended angles. This means a crater twice as big as another will have a subtended angle twice as big; a crater half the size of the Moon will have a subtended angle half that of the Moon. This allows us to set up a ratio of the diameters in different units and solve for the actual diameter of the largest crater in km.

Crater in km / Moon in km = Crater in arcsecs / Moon in arcsecs

3. Calculate the crater diameter.

Activity II: Find the Height of the Peak in the Large Crater

When you look at the image *moon*, where does the sunlight seem to be coming from? Examine the shadows cast by the various features in the image and try to visualize where the sun must be relative to the Moon.

4. Draw a diagram of the positions of the Sun, the Earth and the Moon at the time this image was taken.

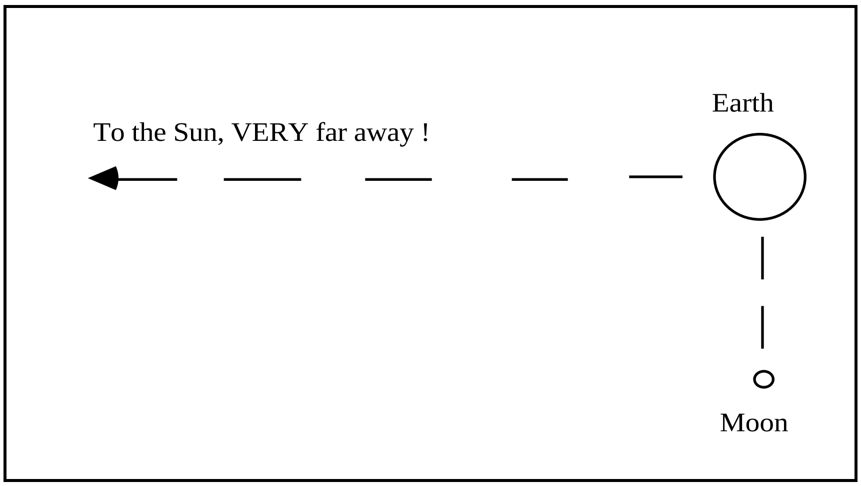
5. On your diagram, estimate the angle between the line of sight from the Earth to the Sun and the line of sight from the Earth to the Moon.

We say that the Moon orbits the Earth, and from the Earth we notice that the appearance of the visible part of the Moon changes from day to day. You are probably familiar with terms such as First Quarter, Last or Third Quarter, New, and Full Moon. These refer to the phases of the Moon at different times during its orbit around the Earth.

6. For the image *moon*, what do you think was the phase of the Moon? Make a good guess if you are unsure.

To verify the actual phase of the Moon on October 17th, 1988, you can look in an ephemeris (an astronomical almanac that contains listings about astronomical objects at different times), or use a computer planetarium program.

The diagram below shows the Sun-Earth-Moon relationship looking down from far above the Earth's North pole when the Moon is in its first quarter phase.



7. For the previous diagram, estimate the angle between the line of sight from the Earth to the Sun and the line of sight from the Earth to the Moon.

When determining the height of a peak by measuring its shadow, *it is important that the Sun makes approximately a 90-degree angle with the line from the Earth to the Moon; i.e., a first or third quarter Moon.* Your angle estimate above should have been 90 degrees or very close.

In the Comparing *Lunar Crater Heights Unit* the shadows of craters are used to compare relative heights of crater walls. You also can use the length of the shadows to determine the actual heights of these or other features.

Finding actual heights of features on the Moon, such as the peak in the center of the large crater, is again a matter of working with ratios.

How to Find the Height of the Peak

Imagine flying in a helicopter over a city when the sun is low in the sky. As you look down on the city from directly overhead, you see the tops of tall buildings and their shadows cast on the ground. Later you land at the airport a few miles away, look back at the city and see the tall buildings silhouetted against the sky.

Now consider standing on Earth and viewing a mountain peak on the surface of the Moon. If the peak is near the terminator, then the sun is low in the Moon sky as seen by someone standing on the Moon near the peak. The terminator is the shadow line between night and day, or light and dark on the Moon. Your view from Earth is analogous to the helicopter view: from Earth you are looking “down” on the Moon from “above.” (See Figure a) You can see Moon features and the shadows cast by those features. Viewing Moon features from the “side,” on the other hand, will be like the silhouettes of the tall buildings in the city after landing at the airport. (See Figure b) In one case you see the apparent length of the shadow, in the other you see the height of the object. The two are related.

This changing perspective helps understand how we can find two similar triangles in order to set up the ratios of corresponding sides and compute the actual height of a mountain peak on the surface of the Moon.

Drawing the Two Triangles

It is important that the terminator be visible in the image. The following figures are NOT drawn to scale, but should help you visualize how we develop the tools we need.

- First, imagine viewing the peak from above and then from the side.
- Get a Styrofoam ball approximately 10 cm in diameter and stick a push pin in the center to simulate the peak in Figure a. Rotate the ball until the pin is on top and you can view the pin from the side. This is an example of the changing perspective we are working with in these figures. If possible, get a very bright light bulb to simulate the Sun and examine the shadow cast by the pin.

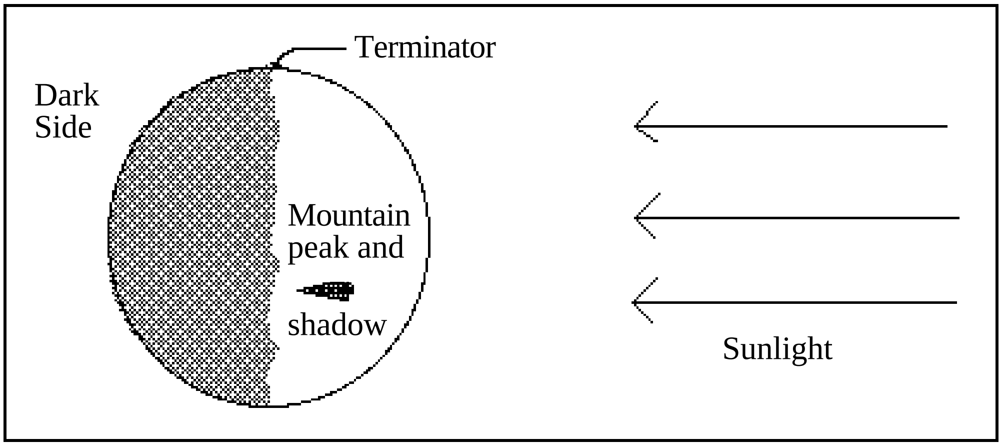


Figure a: The Moon at First Quarter (as seen from Earth)

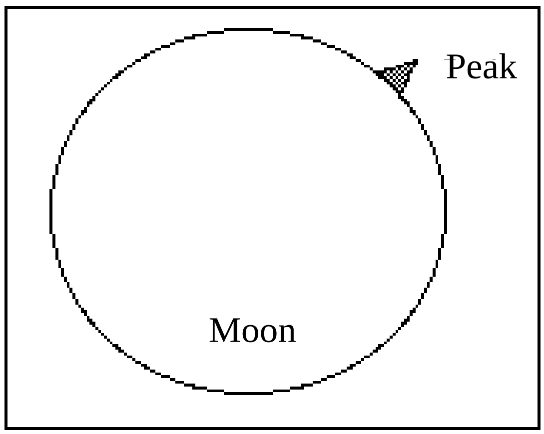


Figure b: The Peak from the Side

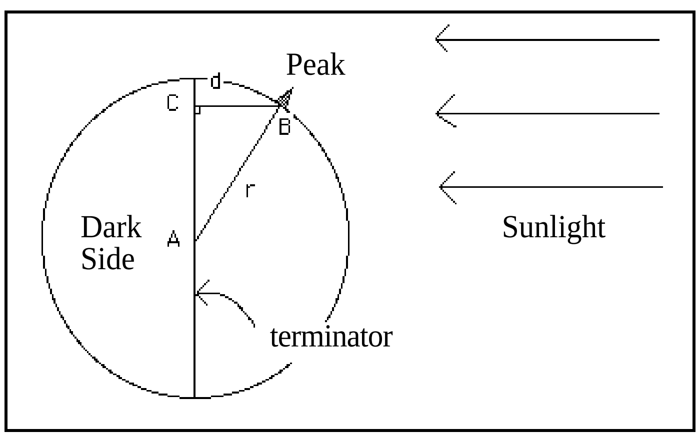


Figure c: Moon triangle ABC

In Figure c, we have constructed a Moon triangle ABC. Point A is at the center of the Moon, point B is at the base of the peak, side r is the radius of the Moon, and side d is the *perpendicular* distance from the base of the peak to the terminator; i.e., line BC.

In Figure d we have redrawn the Moon, viewing the peak from the side as in Figure c, but with an exaggerated peak. BE represents the height of the peak and line s the apparent length of the shadow as we see it from Earth. The center of the Moon is at A and lines AF and AB are radii of the moon. The zoomed drawing of the peak shows DEB as the peak triangle with line BD perpendicular to BE, the height of the peak.

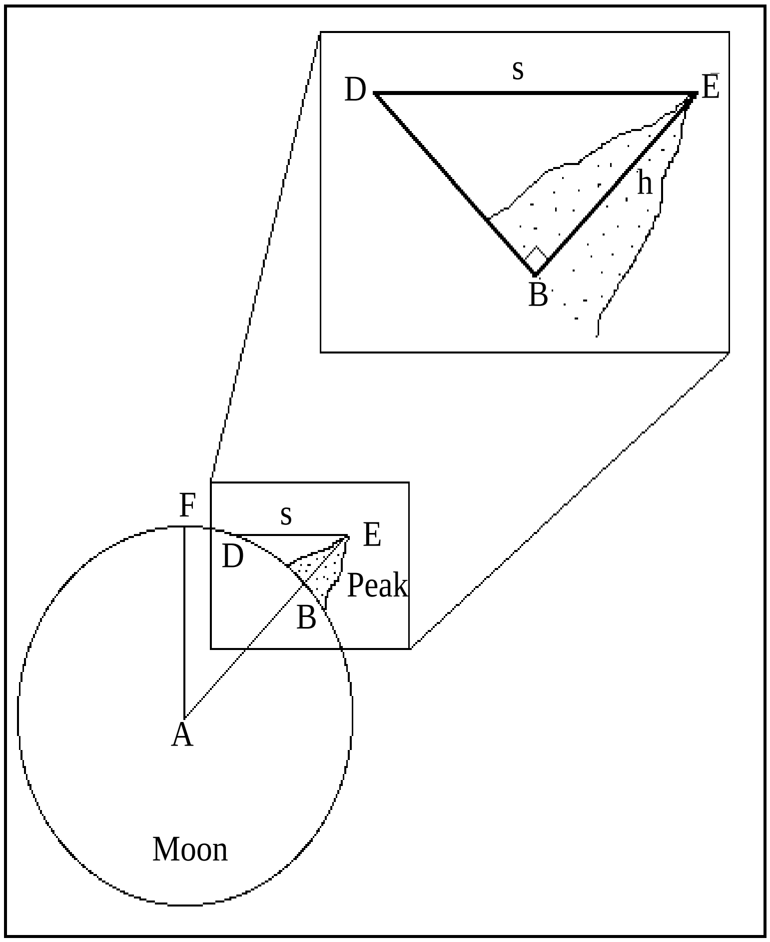


Figure d: Side view of the Peak with "zoom" box
showing the Peak Triangle, DEB

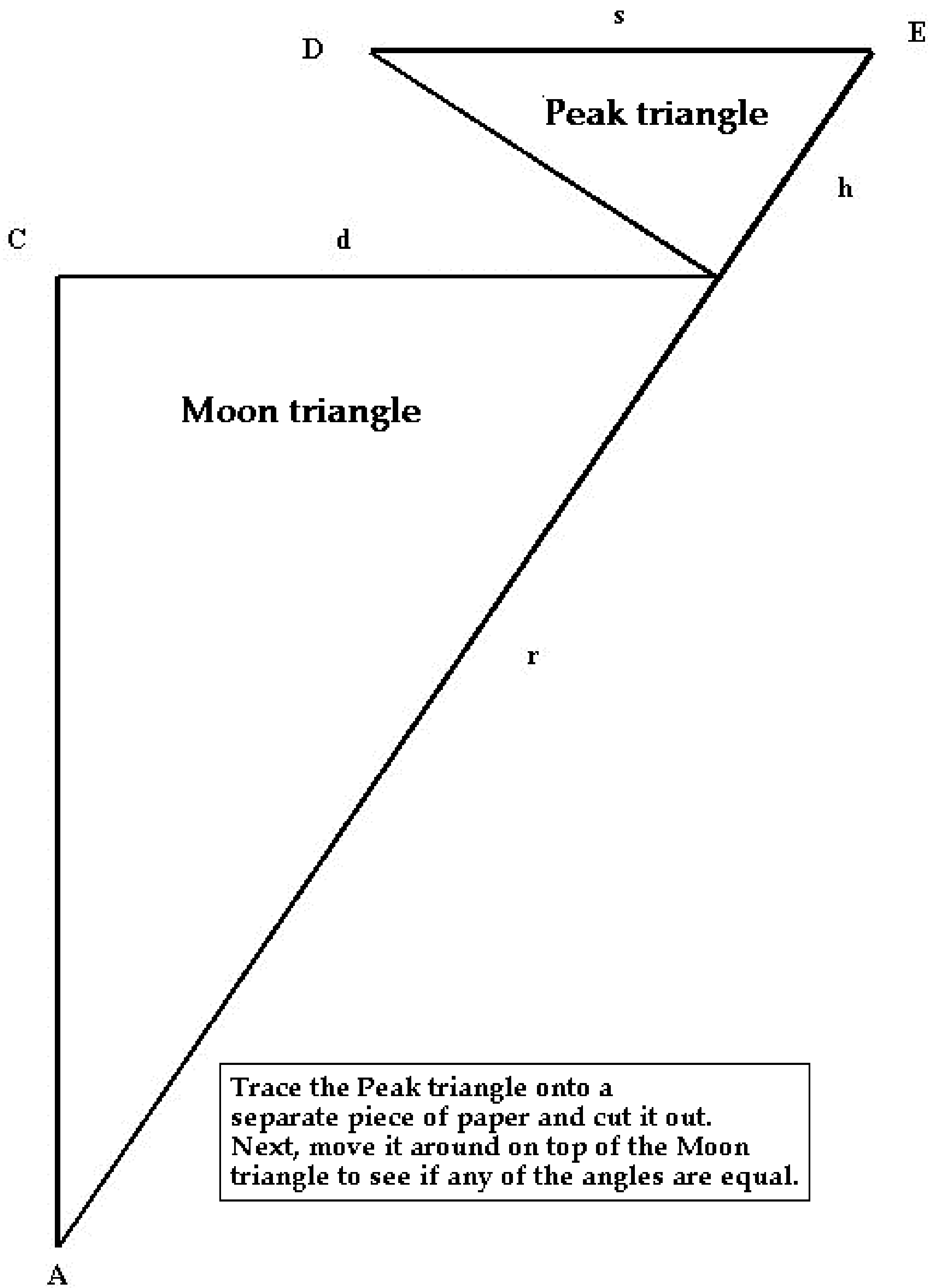


Figure e: Large version of Moon Peak triangles

8. After you traced and cut out the Peak triangle, what were you able to determine about the angles of the two triangles?

9. The Moon triangle and the Peak triangle are similar triangles. The statements that make these triangles similar are given below. Fill in the reasons on the answer sheet in the right hand column. A completed proof is included in the appendix to this unit, but you should try to complete this on your own right now. Spend ten honest minutes before you look at the appendix (either way, you will learn more: proving it yourself or reading a proof after having struggled with the problem). By the way, the proof in the Appendix is not the only possible proof.

Statement	Reason
1. Rays of sunlight are perpendicular to the terminator	(Go to answer sheet to write in reasons.)
2. Side s is parallel to the Sun's rays	
3. Side d is perpendicular to the terminator	
4. Side s || side d	
5. Side h || side r	
6. Angle ABC = angle DEB	
7. Angle ACB = angle DBE	
8. Triangle ABC is similar to triangle DEB	

Finding Values for Three of the Four Sides

The terminator in the *Moon* image is not well defined, as you can see. To find the distance to the terminator, you need to make your best estimate of its location in the image. Remember that you are trying to determine the place on the Moon's surface where the sun is no longer visible.

10. Use Image Processor tools to measure the length in pixels of the peak's apparent shadow, s, and the distance in pixels from the peak to the terminator.

11. For the third side, use the information provided in the Definitions box at the beginning of this unit to convert the Moon radius to pixels.

Calculating the Actual Size of the Peak

12. Estimate how many peaks you could fit end to end inside the crater.

13. Using your knowledge of similar triangles, set up a ratio for corresponding sides of the Moon and Peak triangles and solve for the height of the Peak in pixels.

14. Again use the information provided in the Definitions box at the beginning of this unit. This time determine the number of kilometers (km) in each pixel in the moon image. (This conversion value only applies to images from this telescope.)

15. Use your results from #14 to determine the actual height of the peak.

16. Does your value for the height of the Peak seem reasonable? Explain why or why not.

17. How accurate do you think your value for the height is? Discuss the precision of your measurements and how you dealt with significant figures in your calculation.

18. Using your answer for the diameter of the crater from Activity I and the size for the Peak you found above, calculate how many peaks would fit end to end in the crater.

19. How does this answer compare with your estimate in #12?

20. Explain why side s in Figure d is called the apparent length of the shadow and not just the shadow length?

Locating and Measuring the Large Crater

21. Find the large crater in the *Moon* image on a map or photograph of the Moon. What is its name?

22. Use the scale of the map or photograph to determine the size of the crater.

23. How does this compare to the value you calculated in Activity I?

Activity III: A Quick Tour of Some Other Features

- Look at five other features. Include the highest feature in the image, a small crater's diameter *and* wall height, and a feature that is not a crater.

24. For each of your five features:
(a) describe the feature in words,
(b) give its position in cursor coordinates,
(c) measure its pixel size, and
(d) calculate its actual size.

For distances along the surface use your conversion factor for pixels to kilometers from #14 in Activity II. For heights above the surface, the extra step is required using ratios from similar triangles, as you did in Activity II, before using the conversion factor for pixels to kilometers.

Activity IV: Reflection Upon Your Results

How Does the Topography of the Moon Compare with the Topography on Earth?

25. Compare the size of the large Moon crater with craters here on Earth such as Meteor Crater in Arizona that is 1.2 km wide.

26. Compare the height of the Moon peak with mountains on Earth — from Pikes Peak to Mt. Everest.

27. Calculations of distances on the surface of the Moon are most accurate for features near the center of the Moon as we see it. Why is this so?

Activity V: Follow-up Activities

- The next time the Moon is near first quarter phase, go out with a telescope and look at **your** crater. Look at the Moon at different times of the month and notice the crater's changing appearance. Notice also how some features are more or less visible depending upon how the sunlight is shining on the Moon.
- Request your own Moon image (Refer to the Observer's Guide in the HOU Reference Manual).
- Moon Observation Journal: The next time the Moon is up, observe the Moon for as many nights as you can. Record your observations on paper. Draw a horizon from your observation point and draw the location and appearance in the sky of the Moon for each night. Make your observations at the same time each evening (±15 min.) and from exactly the same spot. Does the Moon rise at the same time and location on the horizon each night?
- Take a field trip to a nearby observatory to view the Moon with a professional telescope. Have your parents take you if the school cannot. *OR* find a local group of amateur astronomers and go to an open observing session.
- Generate a list of named features and challenge your classmates to find them on a lunar map or photo.

Appendix: Proof of Similar Triangles (see Figures c & d)

A Proof:

Statement	Reason
1. Rays of sunlight are perpendicular to the terminator	First quarter Moon
2. Side s is parallel to the Sun's rays	Definition of side s
3. Side d is perpendicular to the terminator	Perpendicular by construction
4. Side s || side d	Perpendicular to the same line, AC
5. Side h || side r	Parts of the same line
6. Angle ABC = angle DEB	Corresponding angles of parallel lines, s & d, cut by a transversal, DE
7. Angle ACB = angle DBE	Both are right angles by construction
8. Triangle ABC is similar to triangle DEB	Two angles of each triangle are =

Date: _____________ Name: _____________________

Answer Sheet

Measuring Size of Moon Features Unit

Activity I

1. Diameter of the largest crater in pixels.

2. Diameter of the largest crater in arc seconds.

3. Actual diameter of the largest crater in km.

Activity II

4. Diagram:

5. Sun-Earth-Moon angle estimate.

6. Phase of the Moon.

7. Sun-Earth-Moon angle estimate.

8. About the angles of the two triangles.

9. Proof of Similar Triangles

Statement	Reason
1. Rays of sunlight are perpendicular to the terminator	
2. Side s is parallel to the Sun's rays	
3. Side d is perpendicular to the terminator	
4. Side s || side d	
5. Side h || side r	
6. Angle ABC = angle DEB	
7. Angle ACB = angle DBE	
8. Triangle ABC is similar to triangle DEB	

10. Distance in pixels of peak's apparent shadow, s, from the peak to the terminator.

11. Moon radius, r, in pixels.

12. Estimate of number of peaks that could fit end to end inside the crater.

13. Height of the Peak in pixels.

14. Number of kilometers (km) in each pixel in the *moon* image.

15. Actual height of the peak.

16. My answer to whether this seems reasonable and why.

17. Comments about the accuracy.

18. How many peaks would fit end to end in the crater.

19. Comparison with my estimate in #**12.**

20. Why side s is called the apparent length of the shadow and not just the shadow length.

21. Name of the large crater.

22. Size of the crater from the map or photograph.

23. How this compares to my value in Activity I.

Activity III

24. Feature #1: Description:

Coordinates: ______ , ______ Pixel size: ___________

Actual size: ______ ______ (include units)

Feature #2: Description:

Coordinates: ______ , ______ Pixel size: ___________

Actual size: ______ ______ (include units)

Feature #3: Description:

Coordinates: ______ , ______ Pixel size: ___________

Actual size: ______ ______ (include units)

Feature #4: Description:

Coordinates: ______ , ______ Pixel size: ___________

Actual size: ______ ______ (include units)

Feature #5: Description:

Coordinates: ______ , ______ Pixel size: ___________

Actual size: ______ ______ (include units)

25. Comparison of the large crater with craters on Earth.

26. Comparison of the large crater with mountains on Earth.

27. Why calculations are most accurate for features near the center of the Moon.

HANDS-ON UNIVERSE™

THE MASS OF JUPITER UNIT

By analyzing images of Jupiter and its moons, you can determine values for the variables D and T in the equation below and solve for the mass of Jupiter, M_J.

$$M_J = \frac{4\pi^2 D^3}{G\ T^2}$$

In this equation, D is the radius of orbit of one of Jupiter's moons and T is the time it takes the moon to complete one orbit (the orbital period). G, the constant of universal gravitation, has a currently accepted value of: $G = 6.67 \times 10^{-11}\ m^3/kg\text{-}sec^2$. This equation applies to any central body that is being orbited by a much less massive object; e.g., The Hubble Space Telescope orbiting the earth, a moon around a planet, one of the planets around the sun, or the sun around the center of our Milky Way galaxy. In all these cases the mass of the orbiting body is insignificant compared to the mass of the central body, and as you can see, its mass is not even included in the equation. If the mass of the orbiting body were significant, they would be orbiting around a common center, and a different equation would be needed.

1. As a practice problem, use the equation above to find the mass of the Earth in kilograms given the following observational data. The period of the Moon around the Earth is 27.3 days and the mean radius of its orbit is 384,000 km. Use meters for the units of D and seconds for T.

Work on one of the Options below as directed by your teacher.

Option A - Your PLAN

Devise a plan for using Jupiter images to determine the mass of Jupiter.
This is not a simple task if you do it thoroughly. It must include a method for determining the radius of the orbit of at least one moon and the period of revolution of that moon around Jupiter.

Option B : This option assumes you have already done the Tracking *Jupiter's Moons Unit.* The following Information Box gives data and conversion factors you will need.

1 degree = 60 arc minutes: 1º = 60'
1 arc minute = 60 arc seconds (arcsecs): 1' = 60"
1 pixel = 0.63 arcsecs (Plate scale for telescope that took this image)
1 radian = 57.3 degrees
Distance of Jupiter from Earth for Jup5 to Jup10 images = 6.63×10^8 km
1 km = 1000 m

Information Box

Determining the mass of Jupiter

a) You need the distance data you determined in the *Tracking Jupiter's Moons Unit* for the images Jup5 through Jup10. Distances need to be in pixels; go back to the *Tracking Jupiter's Moons Unit* and redo this if your original measurements were in millimeters.

b) For each image you also need to know the time of day of the exposure. Use **Image Info** in the **Data Tools** menu. Time is given as Universal Time, UT, which is the time at Greenwich, England. Universal Time is based on a 24-hour clock rather than our familiar 12 hour ones.

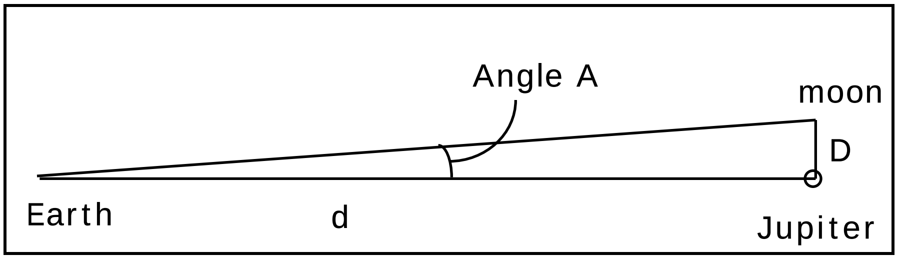
2. Organize your distance and time data in a neat table before you proceed. Call the distances for the moons to the left and below Jupiter in the image negative (-) and the distances to the right and above Jupiter positive (+).

3. For all four moons in all the images, plot the pixel distance from the center of Jupiter versus the time the image was taken.

4. What does the plot you made above represent?

5. Use your plot to estimate the maximum distance for the moon that reaches its turn-around point; i.e., the moon that seems to stop getting further away from Jupiter.

Your pixel distance from question 5 can actually be thought of as the angle subtended by imaginary lines connecting Jupiter and its moon. Line D is the radius of the moon's orbit.



We can use this "pixel angle" to find the radius, D, in km once we convert the pixel value of Angle A to units of radians.

6. Convert the pixel value you found above to radians using the Information Box at the start of this option.

7. Use the Small Angle Approximation (this is derived in Appendix B) to determine the radius of the moon's orbit in kilometers.

D = **d** x **A**   where **D** is the radius of the moon’s orbit, **d** is the distance from Earth to Jupiter at the time the images were taken, and **A** is the angular distance of the moon from Jupiter in radians.

This is the value for one of the two variables you need in order to solve for the mass of Jupiter. To determine the period of the moon, which is the other variable, T, you need to extrapolate from your data by sketching what you think the graph would look like with data for more hours. Use your extrapolation to estimate the time for one quarter of an orbit and for one half an orbit.

8. Use your estimates of time for 1/4 and 1/2 an orbit to determine the period of the moon.

9. Estimate how much possible error there is in your value for the period and explain how you made your error estimate.

10. You now have the period and radius for one of the moons. Use this information to determine the mass of Jupiter from the equation for M_J. Use meters for the units of D and seconds for T.

11. Find a data table and look up the currently accepted value for the mass of Jupiter. Determine the percent difference between the accepted value and your calculated (experimental) value using the following equation:

$$\%\ \text{Difference} = \frac{\text{accepted value - experimental value}}{\text{accepted value}} \times 100\%$$

Wrap Up:

12. Design an experiment that would allow you to obtain a more accurate value for the mass of Jupiter. Be specific.

Date: _____________ Name: _____________________

Answer Sheet

The Mass of Jupiter Unit

1. Earth's mass:

2. Distance and time data:

	Jup5	Jup6	Jup7	Jup8	Jup9	Jup10
Time:	_ _	_ _	_ _	_ _	_ _	_ _
Distance of:						
Moon1	_ _	_ _	_ _	_ _	_ _	_ _
Moon2	_ _	_ _	_ _	_ _	_ _	_ _
Moon3	_ _	_ _	_ _	_ _	_ _	_ _
Moon4	_ _	_ _	_ _	_ _	_ _	_ _

3. Plot of pixel distance of each moon from the center of Jupiter versus the time of day:

4. What the plot represents:

5. Estimate of the maximum distance for the moon that is at its turn-around point:

6. Value in radians:

7. Radius of the moon's orbit in kilometers:

8. Period of the moon:

9. Estimate of % possible error in my value for the period:

How I made this error estimate.

10. Mass of Jupiter from the equation: $M_J = \dfrac{4\pi^2 D^3}{G T^2}$

11. Currently accepted value for the mass of Jupiter:

% Difference =

12. Design for an experiment to obtain a more accurate value for the mass of Jupiter.

Appendix A

Deriving The Mass Equation

In the back of an astronomy text, you will usually find a table called “Planets - Intrinsic Properties” which gives a lot of data about the planets in our solar system. How do we know all these numbers? Specifically, how do we know the mass of Earth and the mass of Jupiter? When we say mass, we are not talking about weight. Mass is a measure of the amount of “stuff” something is made of, the specific bits and pieces (the atoms and molecules). Each bit has a certain mass, usually measured in kilograms. Mass does not change, no matter where in the universe it goes. Our mass on Earth will be exactly the same on Star Trek's Deep Space Nine Station. Weight, on the other hand, is a measure of the force of gravity wherever we might be: on Earth, Jupiter, or in the Andromeda Galaxy. Our weight will change from place to place.

We can determine the mass of Jupiter using the laws that govern the motion of satellites (moons) orbiting a very large planet. All scientific theories and laws are built upon the work of many different people. Our work in determining the mass of Jupiter is no exception. The scientists who have received most of the credit for the rules we will follow are Tycho Brahe (born in 1546 to a Danish noble family), Galileo Galilei (born in 1564 in Pisa, Italy), Johannes Kepler (born in 1571 in Weil, Germany) and Isaac Newton (born in 1642 in England). These people used their own brand of genius along with the data and theories of others to develop laws about planetary motion.

Isaac Newton is possibly most well known for his three laws of motion. Newton's Laws of Motion can be stated as follows:

1. Every object continues in its state of rest, or of motion in a straight line at constant speed, unless forces are exerted upon it to change its state.
2. The acceleration produced by a net force on an object is directly proportional to the magnitude of the net force and inversely proportional to the mass of the body. The acceleration is also in the same direction as the net force.
3. Whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first.

As you can see, according to Newton's first law, any object in motion will keep moving in a straight line forever unless it is acted upon by a net unbalanced force. The moons of Jupiter do not, however, move in straight lines forever; instead they orbit around Jupiter in nearly circular paths. This means there must be a force pulling them towards the center of the planet, called a "centripetal" (center seeking) force.

Here is the equation for the mass of Jupiter: $M_J = \frac{4 \pi^2 D^3}{G T^2}$

In order to derive this you will need the following three equations.

The equation for the magnitude of a centripetal force is: (1) $F = \frac{m_m v^2}{D}$

where m_m is the mass of the moon, v is the speed of the moon in its orbit and D its distance from the center of Jupiter, which is also the radius of its orbit around Jupiter.

In equation (1) we still do not see the symbol for the mass of Jupiter. We do, however, see the variable for the speed of the moon. For circular motion, the equation for speed is:

(2) $v = \frac{2 \pi D}{T}$

where D again represents the radius of the orbit, and T stands for the time for one complete orbit, called the period of revolution.

Isaac Newton, it seems, may have been the first person to surmise that the force that causes objects to fall to the Earth, such as an apple, could be the same force that keeps the moons of Jupiter in orbit. He determined that the force responsible for this motion depends upon the product of the masses of the objects and their distance apart according to the following relationship, known as Newton's Law of Universal Gravitation:

(3) $F = \frac{G M_J m_m}{D^2}$

M_J is the mass of Jupiter, m_m is the mass of the particular moon, and D is the distance of the moon from the center of Jupiter. The symbol G is the gravitational constant and was first determined by Henry Cavendish in 1798.

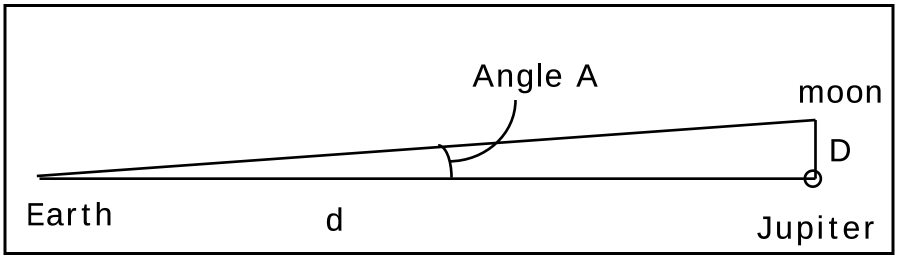
The currently accepted value is: $G = 6.67 \times 10^{-11} \frac{m^3}{kg\text{-}sec^2}$

Given these three equations, you should now be able to derive the equation for the mass of Jupiter: $M_J = \frac{4 \pi^2 D^3}{G T^2}$

Appendix B

A Derivation of the Small Angle Approximation

Examine the diagram below:



Angle **A** is the very small angle between a line of sight from Earth to a moon of Jupiter, and a line of sight from Earth to Jupiter; **D** is the actual distance of the moon from Jupiter, and **d** is the distance of Jupiter from Earth at the time the image was taken. When the angle at Jupiter between lines d and D is 90^{o}, the ratio **D/d** is equal to the tangent of angle **A**.

(1) tan(**A**) = **D/d**

Don't worry about how you will find tan(A) because we are about to figure out a way around that. To find the distance of your moon from Jupiter, you can solve the expression above to get:

(2) **D** = **d** tan(**A**) .

However, since angle A is so small, tan (A) is approximately equal to A when A is measured in radians. Try it with a calculator, but be sure to set it for using radians. Look at values for the tan of angles less than 0.25 radians compared to the angle itself. This means that we can use the following simple relationship, known as the Small Angle Approximation, to determine D :

(3) **D** = **d** x **A** .

(Those of you who have studied pre-calculus mathematics should have already seen the expression s = r theta, where s is arc length, r is the radius, and theta is the angle in radians subtended by s.)

HANDS-ON UNIVERSE™
SUPPLEMENTARY ACTIVITY 8
PLANETS AROUND A PULSAR

The "Science and the Citizen" section of the March 1992 issue of *Scientific American*. included a piece called "Unlikely Places" about evidence of two planets orbiting a pulsar, PSR 1257+12. I got hooked trying to make sense of the article, and my questions led me into a rich panorama of science concepts and processes. Included were false paths and misinterpretations as well as bridges that made things fit and make sense. Which is not to say I ran out of questions.

Since 1992, there have been more discoveries of planets outside our solar system; however, these planets are orbiting stars. The evidence is of the same indirect nature, a tiny wobble. Philip Morrison, in the May 1996 issue of *Scientific American*, estimates there are 10 million such suns in our Milky Way galaxy that illuminate recognizable planets.

The emphasis in this activity is more upon the process of trying to make sense of the 1992 article than upon the "right answer." Explore to see what you can understand and how that exploration connects and uses what you already know. In the process you should find questions you can answer, but also questions you cannot answer. And that is OK. If you are going to engage in research projects, this will be a part of your reality.

There is no given set of questions you should ask, nor any final understandings you should arrive at. For this reason, I hesitate to talk about my own questions, explorations and discoveries since retreading my path is not the point. Given this caveat, however, I will talk about some of the questions I pursued and the places I arrived at as a way of giving a sense of the rich potential here for learning and as an example of the winding yet fruitful path exploratory learning can take.

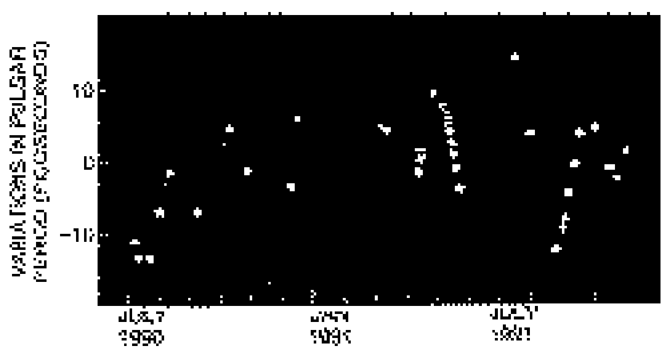
My first probing was prompted by my disbelief that a small wobble back and forth, about 900 km, of a distant pulsar, 1500 light-years away, could be detectable. Since the wobble is caused by the gravitational attraction of the two hypothetical planets, the wobble effect is due to differences in the net force as the two planets move between being in opposition and being in conjunction.

Question #1: Use the information about the periods of the two planets, 98.2 and 66.6 Earth days, to determine the time between one conjunction and the next for the two planets. Is this consistent with the times shown in the *Scientific American* graph below?

Question #2: Use the information about the size of the wobble and the period of the pulsar to calculate the variation in the period of the pulsar this would cause. Is this consistent with the values indicated in the *Scientific American* graph?

Question #3: What about the smaller patterns in the graph? What are they? How can you explain them?

Question #4: Connections: e.g., wave patterns and two pendulums. How is this pulsar's behavior similar to and how is it different from these more familiar phenomena? What are other familiar phenomena that exhibit some of these behaviors?



The existence of pulsar planets is inferred from variations in the pulsar's radio emissions.

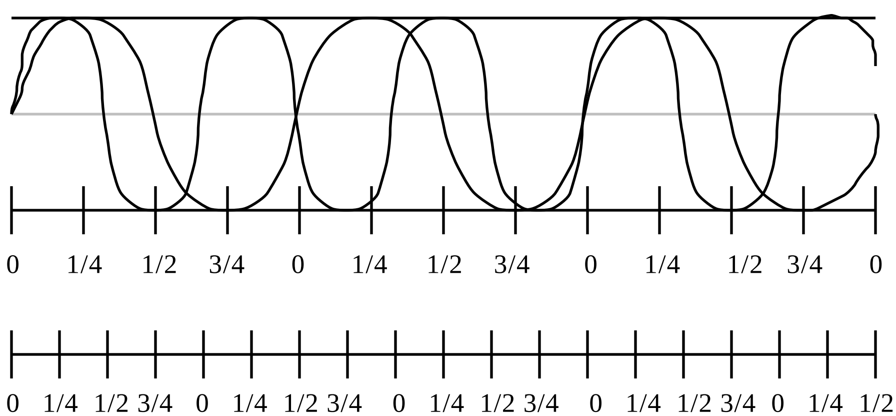
Following is a sample from my notes as I explored questions #1, #3, and #2. It is not shared here as a way to understand the phenomenon but rather as a way to see one person's path as he pursued his own questions. For this reason, it is probably best to stop at this point and pursue your own inquiry into the meaning of the *Scientific American* article and the questions it raises for you before going on to see what I did, and did not do.

/ - - - - - - - - - - /

I started by graphing the motion of the two planets around the pulsar with time along the x-axis and position in the orbit along the y-axis and using a 3:2 ratio for the periods as a reasonable approximation of their given periods: 98.2 and 66.6 Earth days. I then wrote in underneath the graph y-axis values for each of the planets.

98.2 day and 66.6 day cycles ~3:2

When will these be opposed and when together?



add. opp add opp

<------ 2 orbits/3 orbits -------> 6 is LCM, Least Common Multiple

In retrospect there are two confusing things about my graph.

- Writing y-values beneath the x-axis makes that look like the x-axis scale rather than the y-values corresponding to the times along the x-axis.
- The graph looks like a graph of wave reinforcements and cancellations, such as might occur in a ripple tank; in fact, at this point I was thinking of it this way, which accounts for my comment below about the fine pattern not making sense.

From my graphs I saw that the planets lined up again after two and three orbits respectively, and given my approximate 99:66, or 3:2 ratio of periods, that came out to six and a half months. Quoting from my notes:

> "This coincides with the *Scientific American* graph but only for time between Maxs and time between Mins. However, there are two lesser Maxs and two greater Mins between these, and these vary from one cycle to the next.
>
> "The graph above only indicates a smooth cycle without the finer pattern within. Is this the result of going from a 2-D model to 3-D? Not clear why it would."

The next day I came back to this fine pattern question.

> "I realize I was thinking of wave patterns when interpreting my graph for 3:2 cycles. However, gravity is not like crests and troughs. What would have been a better graphic?
>
> "What could cause lesser Max? Min? Less total combined gravitational force, and/or over a smaller time interval, OR!
> 3-D: combined force but at an angle from the telescope sight line."

I drew a diagram of two moons orbiting the pulsar and looked at the positions corresponding to the finer pattern of maximums and minimums. From this I was able to make sense of the patterns between the two largest peaks, both the existence of two, equally spaced, lesser peaks and their relative sizes.

A remaining question is the overall pattern of increase, as if the graph was drawn on a slanted x-axis. I wondered if this could be due to the motion of the Earth around the sun and then realized that was absurd on at least two counts: the distance traveled by the Earth in 6.2 milliseconds is trivial, and the pattern in the graph spans more than six months, which is half an Earth orbit. This remains an unanswered question for me. Further data for 1992 and 1993 would help confirm whether the pattern even exists.

Calculating a value for the variation involved the following steps.

1. Finding the average speed of the pulsar over the 900 km.
2. Using this value, calculating how far it would travel between pulses.
3. Calculating the time light would take to travel this distance; i.e. finding the average variation.
4. My answer: 1.5 picoseconds.

Quoting from my notes:

> " ...variation in the period in the *Scientific American* graph is +12 to -18 picoseconds. Given that my calculation of the average is 1.5 picoseconds, I seem to be in the right ballpark. If fastest speed were 10 x average, I'd be right on."

Here is feedback from another HOU person to an earlier draft: "An activity in which they could see what you're trying to graph is a double pendulum The period is proportional to sqrt (L/g) where L is the length of the string. So have them set up two pendulums on the same bar with lengths in a 9:4 ratio so the periods would have a 3:2 ratio."

An HOU teacher shared initial reports of this planets-around-a-pulsar hypothesis with her 11 & 12th grade Astrophysics class. One of their initial questions was: "Wait a second, how did planets get there? There should to be nothing around an object that has just supernovaed." That is certainly a perceptive question. What questions does their question raise?

