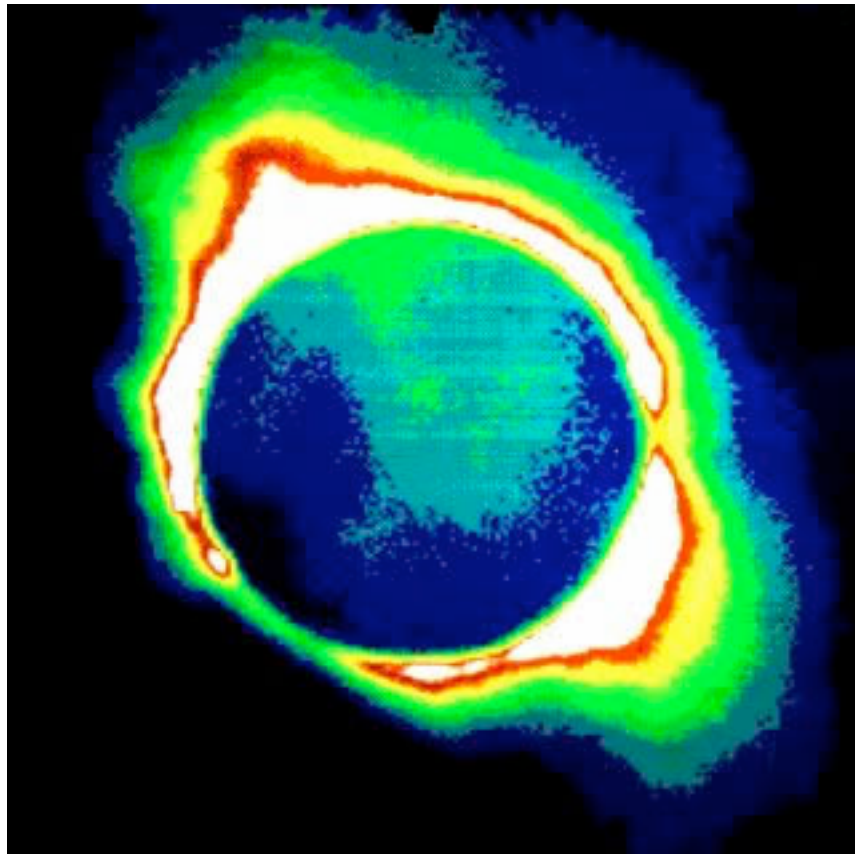


Hands-On Universe/Modeling

Unit VIIIa: Universal Gravitation and Kepler's 3rd Law



Contents

INTRODUCTION.....	3
LABORATORY ACTIVITY 1: QUANTIFYING THE FORCE OF GRAVITY AT THE EARTH'S SURFACE.....	6
WORKSHEET 1: GRAVITATIONAL FORCE AND MASS.....	14
LABORATORY ACTIVITY 2: EFFECT OF DISTANCE ON THE GRAVITATIONAL FORCE.....	18
LABORATORY ACTIVITY 3: CAVENDISH'S EXPERIMENT AND THE DETERMINATION OF G	27
WORKSHEET 2: CAVENDISH'S EXPERIMENT	31
DERIVATION OF KEPLER'S THIRD LAW FROM NEWTON'S LAWS OF MOTION & LAW OF UNIVERSAL GRAVITATION.....	33
WORKSHEET 3: CALCULATING CENTRIPETAL & GRAVITATIONAL FORCES ON THE MOON	36
WORKSHEET 4: CALCULATING THE MASS OF THE SUN	37
LABORATORY ACTIVITY 4: GEOMETRY OF THE JUPITER-MOONS SYSTEM	38
LABORATORY ACTIVITY 5: MASS OF JUPITER LAB	42
WORKSHEET 5: MASS OF MILKY WAY GALAXY CENTRAL BLACK HOLE	45
WS5 TEACHER NOTES: APPLYING KEPLER'S THIRD LAW TO NON-CIRCULAR ELLIPTICAL ORBITS.....	46

Introduction

This unit is the result of work in June 2004 by high school teachers expert in both Hands-On Universe (HOU, <http://lhs.berkeley.edu/hou>, based at University of California, Berkeley) and Modeling Instruction Method (<http://modeling.asu.edu>, based at Arizona State University). The Modeling Method, grounded in *Modeling Theory of Physics Instruction*—educational research by **David Hestenes** and collaborators since 1980, corrects many weaknesses of the traditional science lecture-demonstration method, including fragmentation of knowledge, student passivity, and persistence of naive beliefs about the physical world. The Modeling Method promotes coherence by organizing the course around a small number of scientific models.

Synopsis of the Modeling Method

Coherent Instructional Objectives

- To engage students in understanding the physical world by **constructing and using scientific models** to describe, to explain, to predict and to control physical phenomena.
- To provide students with **basic conceptual tools** for modeling physical objects and processes, especially mathematical, graphical and diagrammatic representations.
- To familiarize students with a small set of basic models as the **content core** of physics.
- To develop insight into the **structure** of scientific knowledge by examining how models fit into theories.
- To show how scientific knowledge is **validated** by engaging students in evaluating scientific models through comparison with empirical data.

Student-Centered Instructional Design

- Instruction is organized into **modeling cycles** [http://modeling.asu.edu/modeling/mod_cycle.html] each of which has two stages: **model development and model deployment**. Roughly speaking, model development encompasses the exploration and invention stages of the learning cycle, while model deployment corresponds to the discovery stage. The two stage modeling cycle has a generic and flexible format which can be adapted to any physics topic. Typically, the cycle is two or three weeks long, with at least a week devoted to each stage, and there are six cycles in a semester, each devoted to a major topic. Each topic is centered on the development and deployment of a well-defined mathematical model, including investigations of empirical implications and general physical principles involved.
- The teacher sets the stage for student activities, typically with a demonstration and class discussion to establish common understanding of a question to be asked of nature. Then, in small groups, students **collaborate** in planning and conducting experiments to answer or clarify the question. Though the teacher sets the goals of instruction and controls the **agenda**, this is done unobtrusively. The teacher assumes the roles of activity facilitator, Socratic inquisitor, and arbiter (more the role of a physics coach than a traditional teacher). To the students, the skilled teacher is transparent, appearing primarily as a facilitator of student goals and agendas.
- Students are required to present and justify their conclusions in oral and/or written form, including a **formulation** of models for the phenomena in question and

evaluation of the models by comparison with data.

- Technical terms and concepts are introduced by the teacher only as they are needed to sharpen models, facilitate modeling activities and improve the quality of discourse.
- The teacher is aware of typical student misconceptions to be addressed as students are induced to articulate, analyze and justify their personal beliefs.

Prerequisites for this unit

Before beginning this cycle, students should be

- a. familiar with the HOU Image Processing software
- b. have previous experience with kinematic models so they have clear concepts of velocity and acceleration.

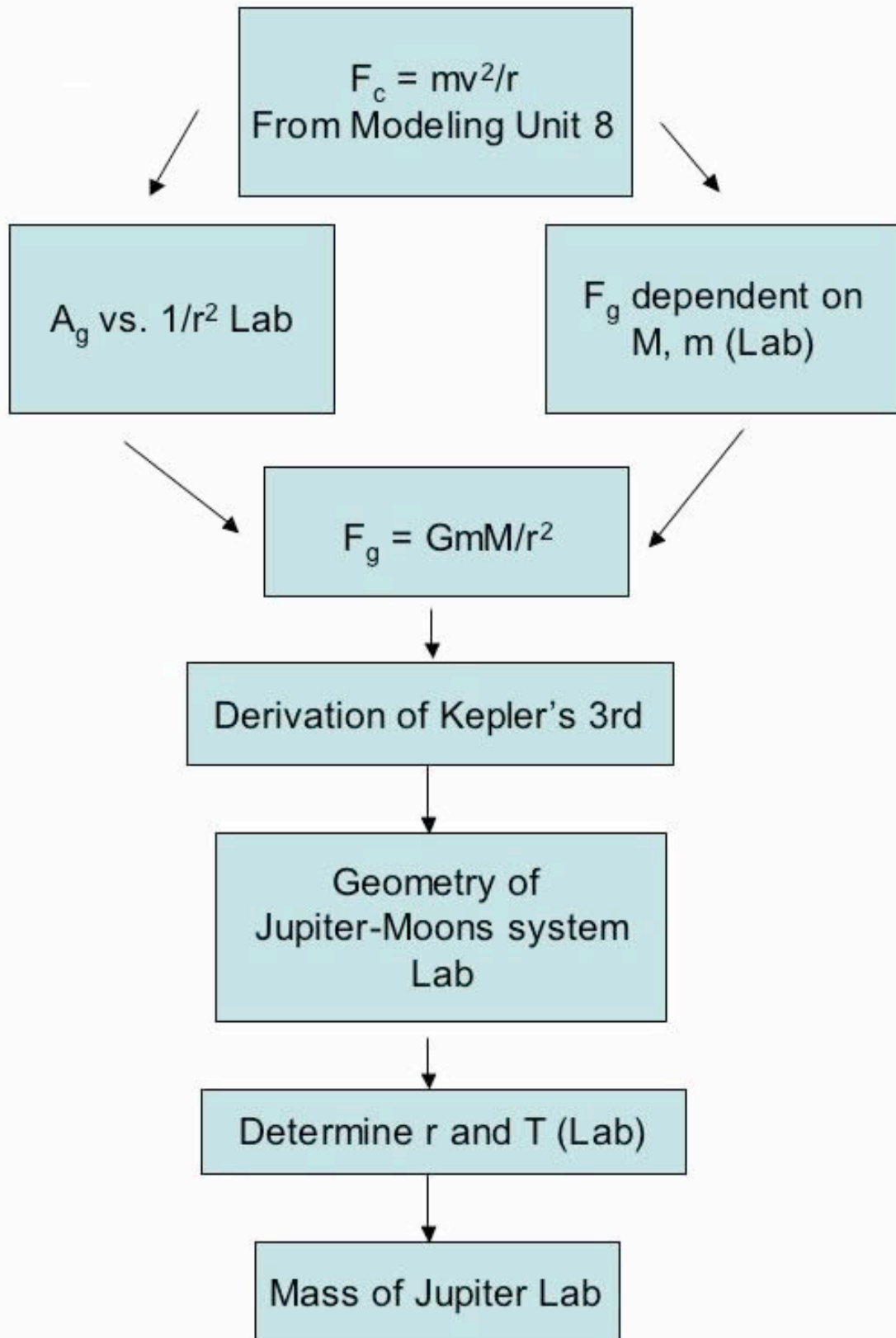
Objectives of this unit

- Discover Newton's Law of Universal Gravitation.
- Derive and apply Kepler's Third Law.
- Understand that the force of gravity is what holds planets and satellites in their orbits, and that this force is the centripetal force.
- Understand the geometry of a parent body-satellite orbital system.
- Determine the orbital period and radius of one moon based on actual images of Jupiter and its moons.
- Determine the mass of Jupiter by using the measured orbital period and radius.

Sequence

Laboratory Activity 1 Quantifying the Force of Gravity at the Earth's Surface
Worksheet 1 Gravitational Force and Mass
Laboratory Activity 2 Effect of Distance on Gravitational Force
Laboratory Activity 3 Cavendish's Experiment and the Determination of G
Worksheet 2 Cavendish's Experiment
Teacher Notes Derivation of Kepler's 3rd Law
Worksheet 3 Calculating Centripetal & Gravitational Forces on the Moon
Worksheet 4 Calculating the Mass of the Sun
Laboratory Activity 4 Geometry of the Jupiter-Moons System
Laboratory Activity 5 HOU "Mass of Jupiter"
Worksheet 5 Milky Way Galaxy Central Black Hole

Concept Map of HOU/Modeling Unit



Unit VIIIa

Laboratory Activity 1: Quantifying The Force of Gravity at the Earth's Surface

Teacher Notes

Note for Modeling Teachers: This is the same lab students did in Unit 4, but with an emphasis on the nature of gravitational forces.

Overview

Add basic idea of $f \propto m$ to mM

Students will use a simple lab involving the use of a spring scale and various masses to determine the gravitational field strength at the surface of the Earth (9.8N/Kg). The influence of varying mass on the strength of gravitational forces is explored as one step toward the ultimate goal of constructing Newton's Law of Universal Gravitation. Follow up labs will provide the opportunity to explore the exact influence of distance on gravitational forces and to calculate the value of the Constant of Universal Gravitation, G .

The lab and the associated extension activities will also provide opportunities to address common misconceptions students often bring to a study of gravity.

An understanding of the nature of gravity and its influence on objects at the Earth's surface can be a springboard to understanding the nature of gravity as a universal phenomenon. Most historical accounts suggest that Newton simply intuited that gravity was a universal force. He reasoned that the force holding the moon in its orbit around the Earth is the same force that causes an apple to fall toward the ground from a tree at the Earth's surface. Students will not necessarily fully accept this view. Other misconceptions about gravity may exist, such as

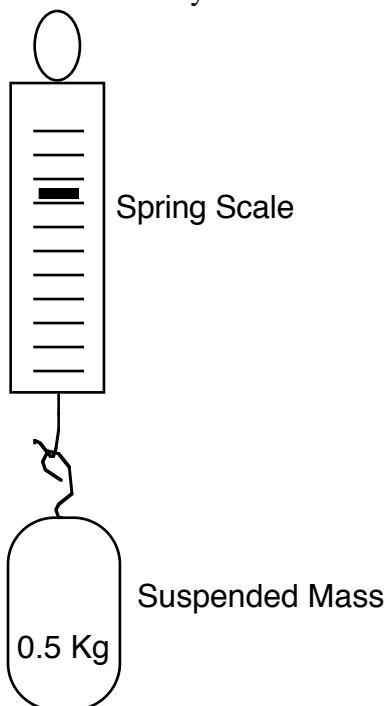
- Gravity does not exist in space (the astronauts in the shuttle are weightless, after all).
- Mass and weight are the same.
- The force of gravity and the acceleration due to gravity describe the same phenomenon.
- The force of gravity is the same for all objects in freefall at the Earth's surface.
- Gravitational forces do not obey Newton's Third Law (i.e. the force of the Earth on a small pebble is greater than the force of the pebble on the Earth).

An opportunity to address these misconceptions is provided by a relatively simple lab designed to quantify the relationship between masses suspended on a spring scale and the force of gravity on those masses.

Pre-lab Discussion

Pre-lab discussion can begin with a simple demonstration such as throwing a ball vertically into the air or dropping an object to the ground and exploring the following questions:

- What happened? Why did this occur?
- What factors influence the strength of the gravitational force in a situation like this?
- How would this event differ if we were to repeat it while standing on the surface of the moon? Why?



Additional background discussion might focus on the concept of force (an interaction between two objects that potentially causes acceleration) and on the difference between weight and mass. These are potentially confusing concepts and explicit dialogue with students about their precise meanings is encouraged. Mass is also commonly defined as the amount of matter in an object (a definition students may bring with them from earlier studies in chemistry), though this definition is somewhat incomplete in the study of mechanics. Mass is best defined as a property of material objects that resists acceleration. Mass is a measure of the amount of inertia an object possesses. Weight is a force and is the result of the gravitational interaction between at least two objects. Weight depends on the mass of the objects in question (the Earth and a person, for example), but mass and weight are not equivalent.

Next, show students a mass suspended on a spring scale. Ask students what they see happening. Record student responses on the board or overhead.

Follow up with the question, what can be measured? Again, record student responses.

Facilitate thinking about the fact that the spring scale measures the force of the Earth pulling on the mass. Ask: Why is the spring stretched? What pulls on the suspended object? A common response to this question is simply “gravity”. However, we suggest insisting on students naming the Earth as the object pulling on the suspended object in order to begin thinking of gravity as a mutual force between two objects, not merely an influence experienced by individual objects at the Earth’s surface. Students should be led to focus on the potential to determine the relationship between the mass of the suspended object (independent variable) and the force of the Earth on that mass (dependent

variable). Naming the force gravity is not necessary at this point, though students will likely freely use the term. In fact, there may be some benefit to referring to the force measured on the spring scale as the “force of the Earth on the suspended object”, since this emphasizes the fact that the spring scale measures the gravitational interaction between the Earth and the suspended object.

Student Lab

Purpose

To determine the graphical and mathematical relationship between the mass of an object suspended on a spring scale and the force of the Earth acting on that mass.

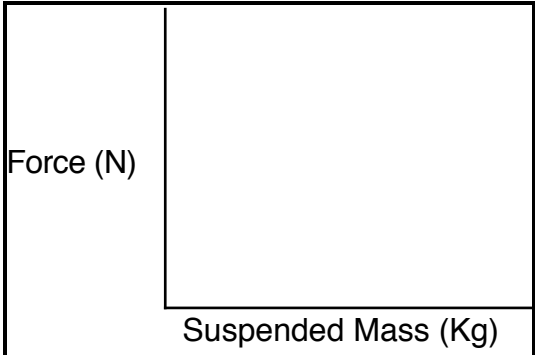
Materials

Spring scales with Newton readings

Varying masses of known mass values

Graph paper or a graphing computer program such as Excel or Graphical Analysis

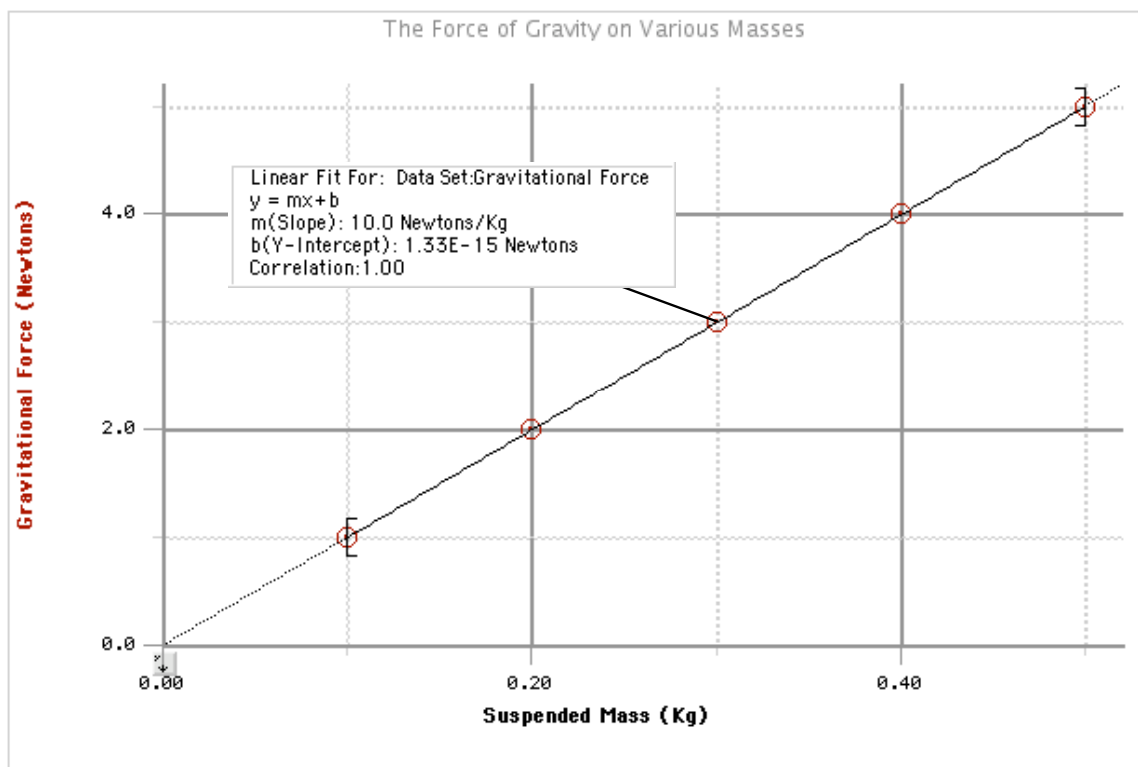
Lab Procedure

<p>In groups of no more than three, students measure the force of gravity acting on at least five known masses. The five data points can be graphed as indicated below. The mass of the suspended object in Kg is the independent variable and the force of the Earth on the object in Newtons is the dependent variable. Sample data is shown below.</p>	
---	---

Data Collection

Mass of Suspended Object (Kg)	Force of Earth on Object (Newtons)
0.1	1
0.2	2
0.3	3
0.4	4
0.5	5

Data Analysis



The best-fit line for the sample data set suggests a slope of 10 N/Kg. This slope represents the “gravitational field strength” at the surface of the Earth. Students will likely come up with a value of 10 N/Kg, given that the precision of most spring scales is not sufficient to allow for more precise values. They can be encouraged to take readings to one uncertain digit, which may yield slopes closer to the actual 9.8 N/Kg.

The mathematical equation derived from the graph can be written

$$F_g = \left(10 \frac{\text{N}}{\text{Kg}}\right)m$$

Post-lab Discussion during Whiteboarding

Questions to address during post-lab discussion:

- What does the graph of the data suggest about the relationship between mass and gravitational force?

As the mass of the object increases, the gravitational force increases as well. The force of gravity is directly proportional to the mass of the object.

Doubling the mass results in a doubling of the force of gravity on that mass. Tripling the mass of the objects results in a force of gravity that is three times larger, and so on.

- What would the force of gravity be on 10 Kg mass at the surface of the Earth?
How do we know?

100 Newtons, based on the equation generated by the data.

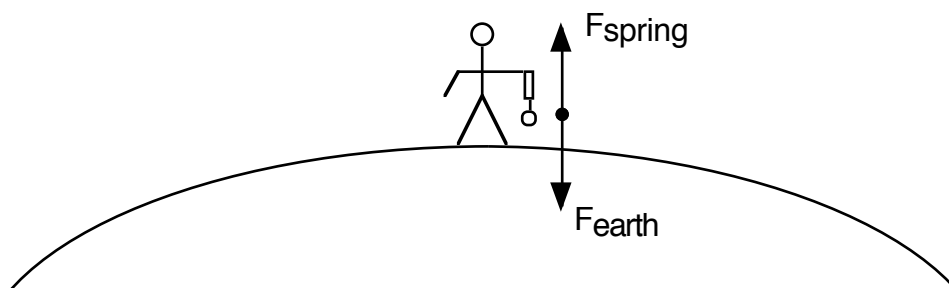
- What does the slope of the graph represent? Explain the slope in words. Note: the term “gravitational field strength” does not have to be used here, and, in fact, probably should not be used until students have fully explained the conceptual meaning of the slope.

The slope represents the strength of the gravitational force on every kilogram at the Earth’s surface. For every 1 Kg of mass, the Earth pulls with a force of approximately 10 Newtons.

- What would you expect a similar graph to look like if data were collected on the moon? Why?

The same experiment on the moon would result in a graph that has a smaller slope since the force of gravity on each Kg at the moon’s surface would not be as great as the force of gravity on each Kg at the Earth’s surface.

- Draw a force diagram for the situation. What is the source of the force acting on the suspended mass?



Including at least a portion of the Earth in the drawing can help students to visualize the force of gravity as an interaction between the Earth and the suspended mass. The forces drawn above can be quantified for any specific mass.

- How many objects must be present for a gravitational force to exist? In the interaction

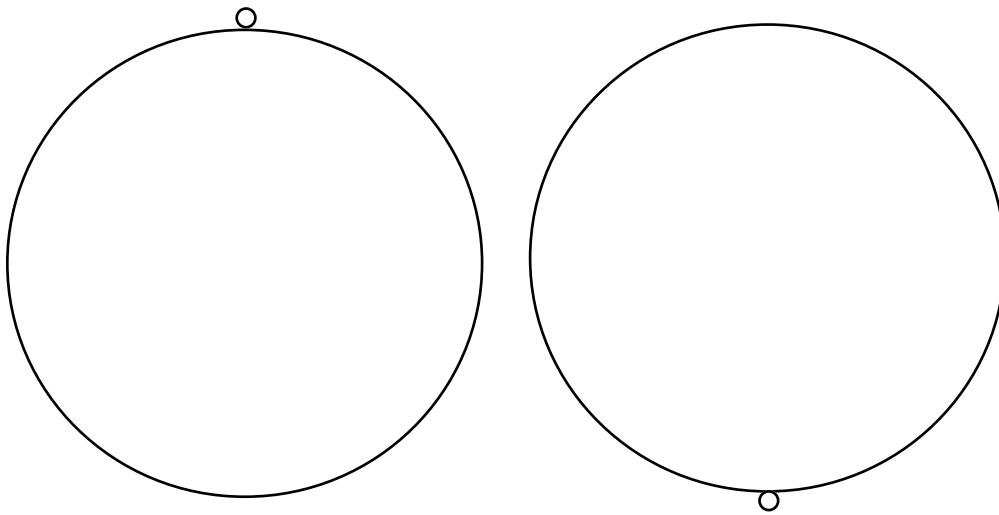
between the Earth and the suspended mass, which object exerts the greater force? Why?

Two objects are required for a force to exist. Newton's Third Law predicts that any force is equal and opposite for the two objects involved. Here the two objects are the suspended object and the Earth. The force of the Earth on the suspended object is equal to the force of the suspended object on the Earth.

Students should be encouraged to consider this concept carefully. A possible line of questioning might include the following:

What is the weight of a 5 Kg bowling ball at the surface of the Earth? (50 Newtons)
What is the weight of the Earth on the surface of a 5 Kg bowling ball? (Also 50 Newtons).

A clear understanding of why the answer is the same requires students to truly understand a number of easily confused concepts. Similar questions are given in the follow up worksheet for this lab (Worksheet 1).



•What can we conclude about the influence of mass on gravitational force?

The force of gravity is the result of an interaction between (at least) two masses and is influenced equally by the mass of each object. The force experienced by the two objects is equal.

Put into question form like above. XXXXXXXXXXXXXXXX

The slope of the line representing the data is a measure of the **gravitational field strength** at the surface of the Earth. Students will likely recognize the similarity between this value and the **acceleration due to gravity** previously studied (9.8 m/sec/sec). We

suggest being very explicit in making a distinction between these two values. The strength of Earth's gravitational field, g , can be used to determine the magnitude of the gravitational force on a specific mass ($F_g = mg$) whether the object is in motion or not.

The acceleration due to gravity is the *motion* resulting from the force of gravity acting on a particular mass. The gravitational field strength determines the gravitational *force* on a particular mass, while the acceleration due to gravity is the *motion* caused by that force. The fact that these two phenomena have the same value is the result of how the Newton was defined (the force required to accelerate 1 Kg at 1 m/s/s). The distinction between the strength of a gravitational field and the acceleration due to a gravitational force is often overlooked in introductory physics courses. However, explicit distinctions can help students to arrive at a deeper understanding of central concepts in mechanics and will provide a basis for a clearer understanding of the concepts of electric field, electric force, and electric potential studied later in the course.

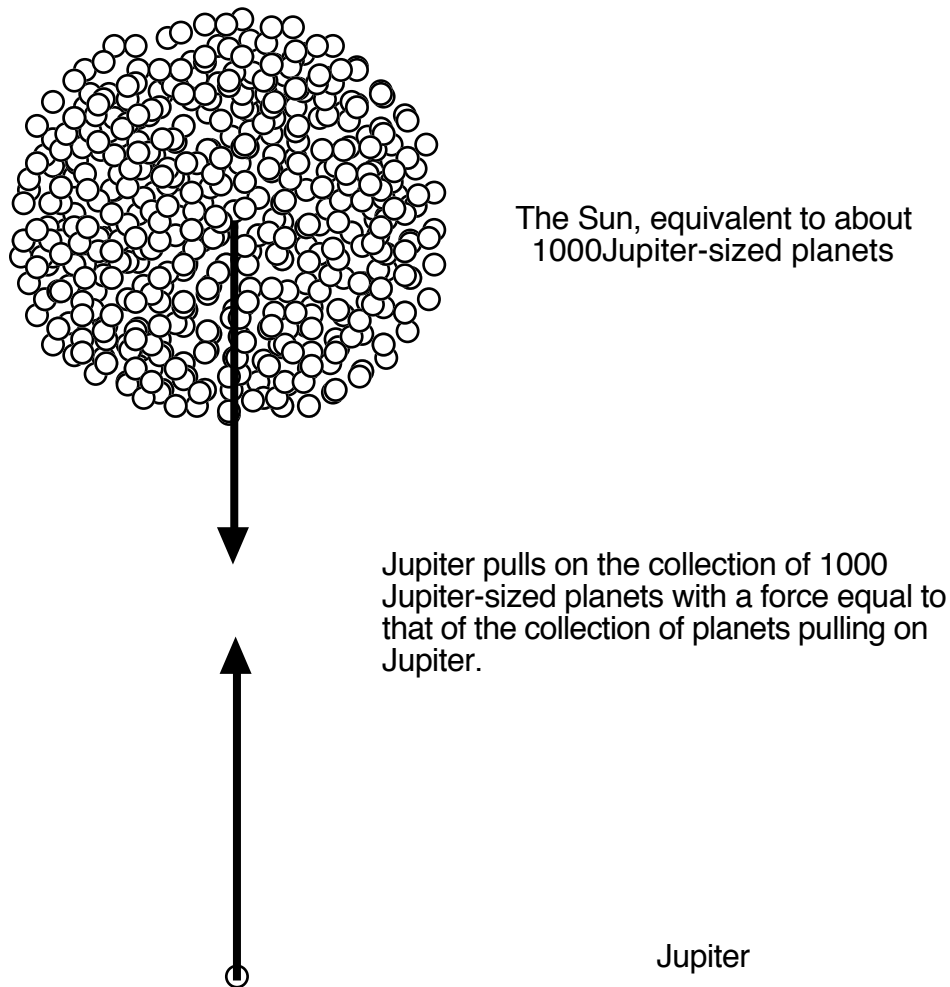
It can be helpful for students to begin thinking about the reading on the spring scale as a measure of the force of the Earth on the suspended object. Gravitational forces are the result of an interaction between two masses, and the resulting force is equal in magnitude for each of the objects involved regardless of their individual mass. Most students initially state that the Earth's force on the mass is much greater than the mass's force on the Earth. This misunderstanding can be addressed by having students draw the Earth in force diagrams in which gravity is present. Also, by reversing the usual question of an object's weight on the Earth and inviting students to consider the Earth's weight on a much smaller object provides an opportunity to think differently about the nature of gravitational forces in everyday circumstances.

In this lab it is obvious that the gravitational force depends on the mass of the suspended object. Students may not recognize that the mass of the suspended object is, in fact, just as important in establishing the gravitational force as the Earth is. Without the suspended object, a gravitational **field** would still be present, but the force itself requires the presence of two masses. Discussion of this point can help students to begin thinking of gravitation as a universal phenomenon that results from *any* mass *anywhere*.

From the lab and post-lab discussion, students should understand that gravitational forces are the result of interaction between at least two masses and that the force present is equal for both masses. The linear relationship between mass and gravitational force given by the lab data suggests that double the mass of one of the objects doubles the gravitational force, tripling the mass causes the gravitational force to triple, and so on. These insights should provide a basis on which students can understand Newton's logic in assuming that gravitational forces depend on the product of the two masses present. He reasoned that if his Third Law is true, then the gravitational forces acting on both bodies must be equal and opposite. Newton used the sun and Jupiter to help explore this relationship further. If the sun, being approximately 1000 times as massive as Jupiter, is thought of as a collection of 1000 Jupiter-sized planets, then each of the planets in the collection pulls on Jupiter with a particular gravitational force, F . The combined effect of each of the 1000 Jupiter-sized planets pulling together is 1000 F . Similarly, Jupiter as an individual planet pulls on each of the 1000 Jupiter-sized planets in the collection with a force of F . Again,

the total force is 1000 F. The only mathematically logical application of this idea is to assume that the gravitational force is proportional to the product of the masses

$$F \propto m_1 m_2$$



Adapted from Project Physics.

Understanding the subtle difference between inertial mass (mass that resists accelerating in the presence of a net force) and gravitational mass (mass that interacts gravitationally with one or more other masses) can be important but can be explored later in the unit after students have a deeper understanding of the nature of gravity. This is a distinction many teachers may decide to avoid, but explicit exploration of these concepts may be appropriate.

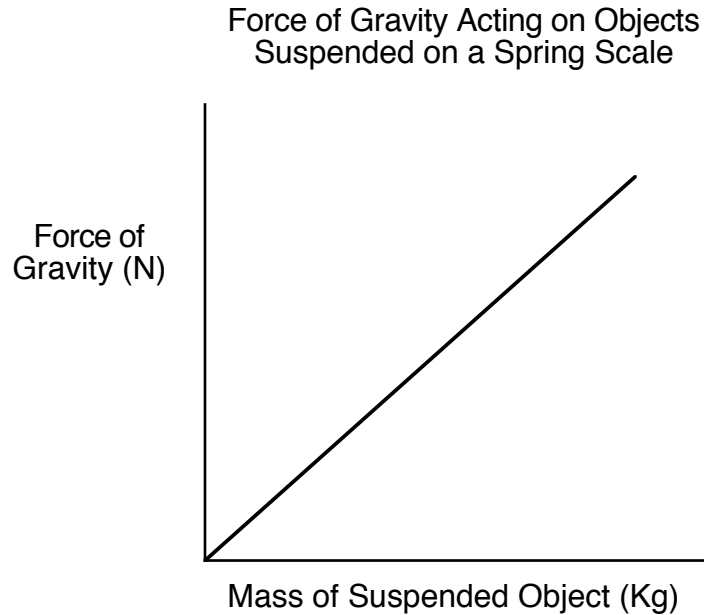
Name _____ Date _____ Per. _____

Unit VIIIa:

Worksheet 1

1. This qualitative graph represents the relationship between the mass of various objects and the gravitational force acting on those objects. The best fit line represents data collected at the Earth's surface.

a. Sketch and label the best fit line that would result if the same experiment were conducted on the surface of the moon. Explain your reasoning fully.

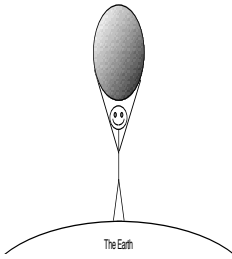


b. Sketch and label the best fit line that would result if the same experiment were conducted on the surface of the sun. Explain your reasoning fully.

2. a. The strength of the gravitational field at the Earth's surface is approximately 9.8 N/Kg. Explain fully what this means.

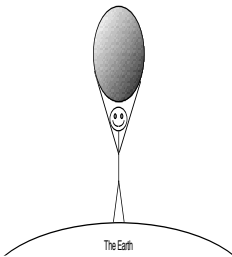
b. The acceleration due to gravity at the Earth's surface is approximately 9.8 m/sec/sec . Explain fully what this means.

3. The diagram below represents the Earth-moon system. Draw qualitative vectors to represent any forces that are present. The relative size of any forces should be evident in the lengths of the vectors drawn.



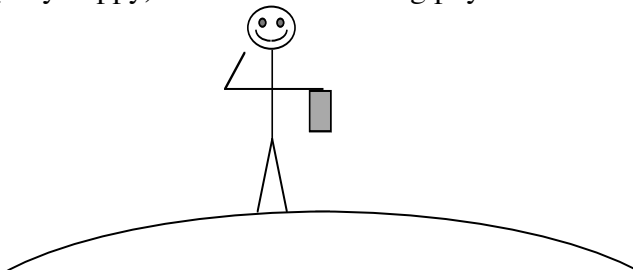
Explain your diagram.

4. The diagram below represents a 2 Kg bowling ball on the surface of the Earth (obviously not to scale).



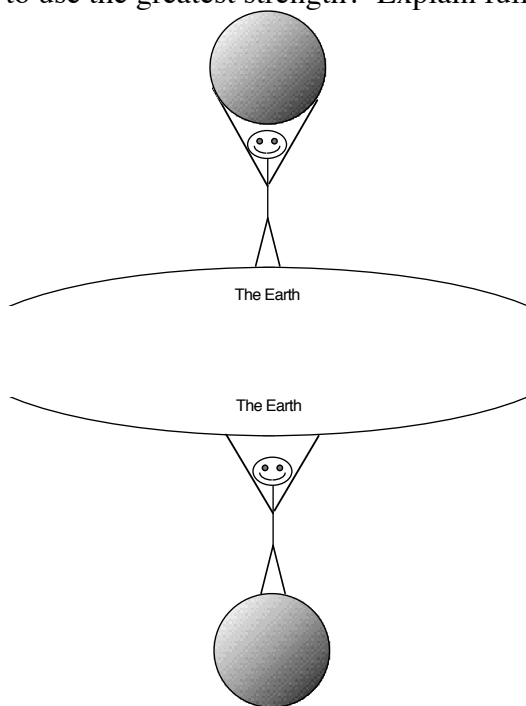
- a. Calculate the force of gravity acting on the bowling ball. Show all steps of your calculation, including the appropriate equation and units.
- b. What is the weight of the bowling ball? Explain.
- c. Assume that the Earth and the bowling ball are the only objects present. How much does the Earth weigh in the presence of the bowling ball? (A question for thought: Is the bowling ball on the surface of the Earth, or is the Earth on the surface of the bowling ball?) Explain.
- d. Recall Newton's Third Law of Motion (action-reaction). If we consider the force of the Earth acting on the bowling ball to be the action, what is the reaction? Explain.

4. A (very happy) student holds a 1 Kg physics textbook at the surface of the Earth.



- a. Draw a quantitative force diagram for the textbook. Clearly label all forces.
- b. What is the force of the textbook on the Earth? Explain.

6. Under which of the following circumstances would Atlas the mighty physics student need to use the greatest strength? Explain fully.



Unit VIIIa

Laboratory Activity 2: Effect of Distance on the Gravitational Force

Teacher Notes

Overview

Prior to doing this lab, students should have studied circular motion (Modeling Unit 8). They should be familiar with the concepts of centripetal acceleration and centripetal force. They should understand that centripetal force is not a separate force, it is just a name for the net force on an object when the net force happens to point to the center of a circle. This lab, the second in this series, will provide a part of Newton's Law of Universal Gravitation. The first lab showed students that the gravitational force is proportional to both masses involved. This lab will allow them to discover that the gravitational force varies with the inverse square of the distance between the two bodies. This lab will also reinforce the idea that the "big" mass also influences the gravitational force. The third lab will introduce the value of the Constant of Universal Gravitation, G .

Pre-lab discussion

In order to get the students thinking about orbiting bodies, show the orbit simulation at <http://observe.arc.nasa.gov/nasa/education/reference/orbits/orbit3.html> ("reference" is spelled "referance" in the address) or open the provided file Orbit3.htm. Ask students what they observe. Make a list of all suggestions they give and write them on the board. Students should also write this list in their lab notebook. (They should easily be able to see that when the satellite is closer to Earth, it moves faster, and when the satellite is farther away it moves slower.) After they have exhausted all responses, ask them what factors could possibly influence the motion of the satellite. If they seem stuck, remind them to think of the concepts they previously studied in circular motion. You are looking for suggestions such as radius of orbit, mass of satellite and Earth, period of orbit, speed of satellite, acceleration of satellite, force acting on satellite, etc...

From the given list, ask students what factors that are measurable (not ones that can be calculated, only direct measurements). Draw a line through any observations which are not measurable. Variables that should be left on the list are mass, period, and radius (velocity, acceleration, and force are values we can calculate from the measurable quantities). Thinking back to their original observation that the satellite moved faster when it was closer to the Earth, have them come up with a purpose for this lab.

Purpose: *How does the distance away from the parent body affect the velocity of a satellite?*

Prediction: *As the distance from the parent body increases, the velocity will decrease. (or however students choose to express their ideas)*

They will need to use some of the variables from their list in order to answer their purpose question. Using the variables they have left on their list, ask students to make a graph of the acceleration of an orbiting body vs. the distance between that body and the parent body. They will need to recognize that the acceleration we are speaking of is centripetal acceleration (do not tell them this). The students should decide what data from their list is necessary to make this graph. (They will need the radius and orbital period in order to complete this task, although do not tell them this.) Ideally, students should research this data from books, the internet, or the orbit simulator. If time or materials do not allow for this, you can provide students a copy of an information table, but be sure to include many variables, not just radius and period, so that the students still have to decide on their own what to measure.

XXXX INCLUDE SAMPLE DATA TABLES FOR TEACHERS IN AN APPENDIX

**** Note:** Usually, to conduct a lab you must choose an independent and dependent variable, controlling all others. In this case, we have chosen to use radius and period, so the mass of the satellite and parent body should be controlled. The mass of the parent body will not change, however all of the satellites have different masses. In this case, it does not matter because the value of acceleration does not depend upon mass (like the acceleration due to gravity). We suggest not bringing this up to the students, as it will likely cause confusion. They will probably not even ask about the mass of the satellite, but if they do, be ready to explain why it does not matter. Tell them that at the end of the Lab 3, you will be able to show them clearly why it does not matter what the mass of the satellite is.

In order to show the relevance of the mass of the parent body, you are going to want this graph for multiple parent bodies (ask students not to use Jupiter, as we are going to do a separate experiment on Jupiter later in the unit). Since the process of calculating and converting all of the numbers is very time consuming, it is best to have the class share some data. Each group should select a different parent body (ideally 2 groups would do the same one, so the total for the class would be 4-5 parent bodies), and then share data so that the class as a whole has 4-5 graphs. You will also need this data to use in Lab 3 of this unit.

Materials

- Data on parent bodies and satellites
- Graphical Analysis

Procedure

After students have thought about what quantities are necessary to make their graph, they need to write a procedure. Since there are not any manipulatives in this experiment, students should write the procedure of what data they need and how they are going to gather it, and how they are going to manipulate that data in order to find acceleration for the graph.

The students reasoning should look something like this: The satellite is traveling in a circular path, and therefore requires a center-pointing force to hold it in its circular path. If there were no center-pointing force, the satellite would fly off in a straight line path. The only force acting on the satellite is the force of gravity, pointing directly toward the center of the Earth (or other parent body). Centripetal force is defined as the net force that points toward the center of the circle. In this case, the only force acting on the satellite is the force of gravity, and it points toward the center of the circle. Therefore, this force is the centripetal force. If there is a centripetal force, there must be a centripetal acceleration (this is what we want to graph). This can be calculated by $a = v^2/r$. The velocity can be found simply by $v = \text{circumference}/\text{period} = 2\pi r/T$. In order to create the graph of acceleration vs. radius, you therefore have to know the radius and period for the satellite.

A sample procedure is below. Make sure to check students' procedures before allowing them to continue, paying special attention to the variables they are using and the units on the numbers. Remind them to use the standard units of meters and seconds. Remember, students should come up with this procedure on their own, not use this one.

Procedure:

1. Choose a parent body (ex: Earth, Sun, Saturn, etc....)
2. Choose 5 satellites (or moons) of the parent body. (If they choose the Sun, only take data for the first 4 planets, so that the range of data is not too large to graph).
3. Find the radius for each satellite (making sure to measure from the center of the satellite to the center of the parent body) and record in data table. Make sure the numbers have the units of meters.
4. Find the orbital period for each satellite and record in data table. Make sure numbers have the units of seconds.
5. Calculate the velocity of each satellite by using $v = 2\pi r/T$. Be sure that the units are m/s. Record these values in a new data table.
6. Calculate the acceleration of each satellite by using $a = v^2/r$. Be sure that the units are in m/s^2 .
7. Graph a vs. r using Graphical Analysis (teacher: see below for instructions and hints on graphing).

Data Collection

(Remember, each group will have data for only one parent body, but as a class, there should be data for 4-5 parent bodies.)

Earth:

Satellite	<i>Radius (m)</i>	<i>Period (s)</i>
Shuttle	$0.662 \cdot 10^7$	5400
Satellite 1	$1.285 \cdot 10^7$	14520
Satellite 2	$2.891 \cdot 10^7$	49020
Satellite 3	$3.534 \cdot 10^7$	66240
Geosynchronous Satellite	$4.216 \cdot 10^7$	86160

Sun:

<i>Satellite</i>	<i>Radius (m)</i>	<i>Period (s)</i>
Mercury	$579 \cdot 10^8$	$0.07560 \cdot 10^8$
Venus	$1082 \cdot 10^8$	$0.1941 \cdot 10^8$
Earth	$1499 \cdot 10^8$	$0.3156 \cdot 10^8$
Mars	$2279 \cdot 10^8$	$0.5936 \cdot 10^8$

Uranus:

<i>Satellite</i>	<i>Radius (m)</i>	<i>Period (s)</i>
Miranda	$1.298 \cdot 10^5$	$1.222 \cdot 10^5$
Ariel	$1.908 \cdot 10^5$	$2.177 \cdot 10^5$
Umbriel	$2.658 \cdot 10^5$	$3.580 \cdot 10^5$
Titania	$4.361 \cdot 10^5$	$7.522 \cdot 10^5$
Oberon	$5.831 \cdot 10^5$	$11.63 \cdot 10^5$

Data Analysis

Earth:

Satellite	Radius (m)	Velocity (m/s)	Acceleration (m/s ²)
Shuttle	0.662 10 ⁷	7.699 10 ³	8.954
Satellite 1	1.285 10 ⁷	5.558 10 ³	2.404
Satellite 2	2.891 10 ⁷	3.704 10 ³	0.4746
Satellite 3	3.534 10 ⁷	3.351 10 ³	0.3178
Geosynchronous Satellite	4.216 10 ⁷	3.073 10 ³	0.2240

Sun:

Satellite	Radius (m)	Velocity (m/s)	Acceleration (m/s ²)
Mercury	579 10 ⁸	4.810 10 ⁴	0.03979
Venus	1082 10 ⁸	3.501 10 ⁴	0.01133
Earth	1499 10 ⁸	2.983 10 ⁴	0.005936
Mars	2279 10 ⁸	2.411 10 ⁴	0.002551

Uranus:

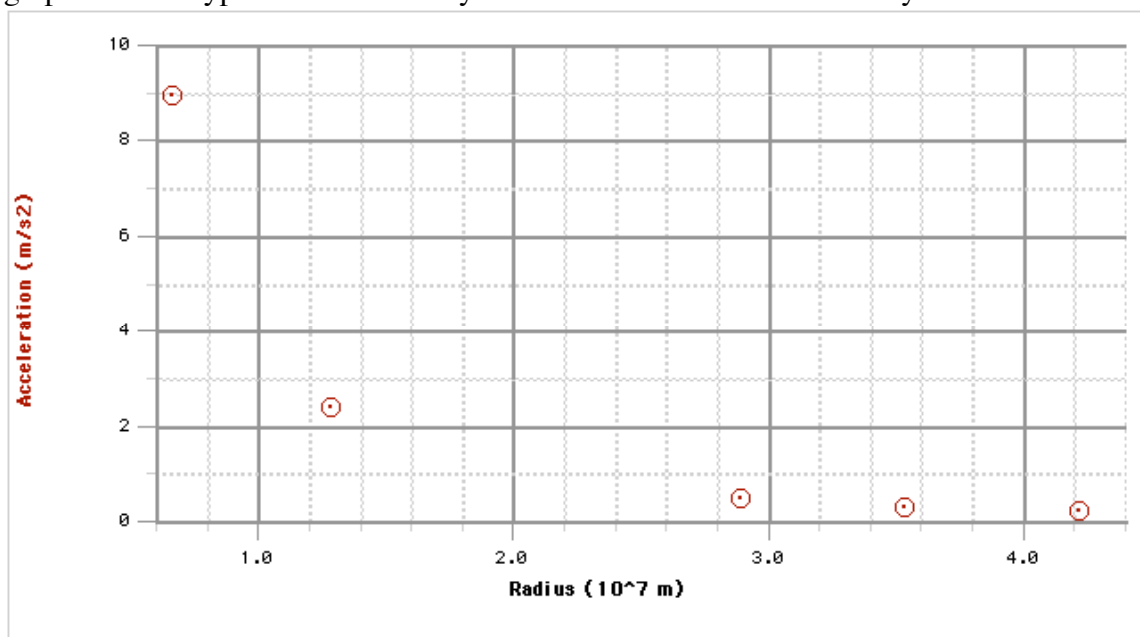
Satellite	Radius (m)	Velocity (m/s)	Acceleration (m/s ²)
Miranda	1.298 10 ⁸	6.676 10 ³	0.3431
Ariel	1.908 10 ⁸	5.504 10 ³	0.1588
Umbriel	2.658 10 ⁸	4.663 10 ³	0.08180
Titania	4.361 10 ⁸	3.591 10 ³	0.02998
Oberon	5.831 10 ⁸	3.149 10 ³	0.01701

****Helpful Hints for Graphing:**

One of the easiest programs for graphing is *Graphical Analysis* by Vernier Software (www.vernier.com). You can obtain a site license for this program for just \$80. When graphing the data, start by graphing a vs. r . Since the numbers are so large, the easiest thing to do is to express all radii to the same power of 10. (See above data tables for an example.) When you enter the radii, enter only the mantissa, not the exponent. Enter the exponent value in with the units (ex: 10⁸ m). Enter the acceleration data without exponents at all. Then when analyzing the slope, you can incorporate the exponent back into the number.

Data Analysis (continued)

Once you have a graph of a vs. r , students should recognize the general shape of the graph as some type of inverse. They should hand sketch the curve they see.



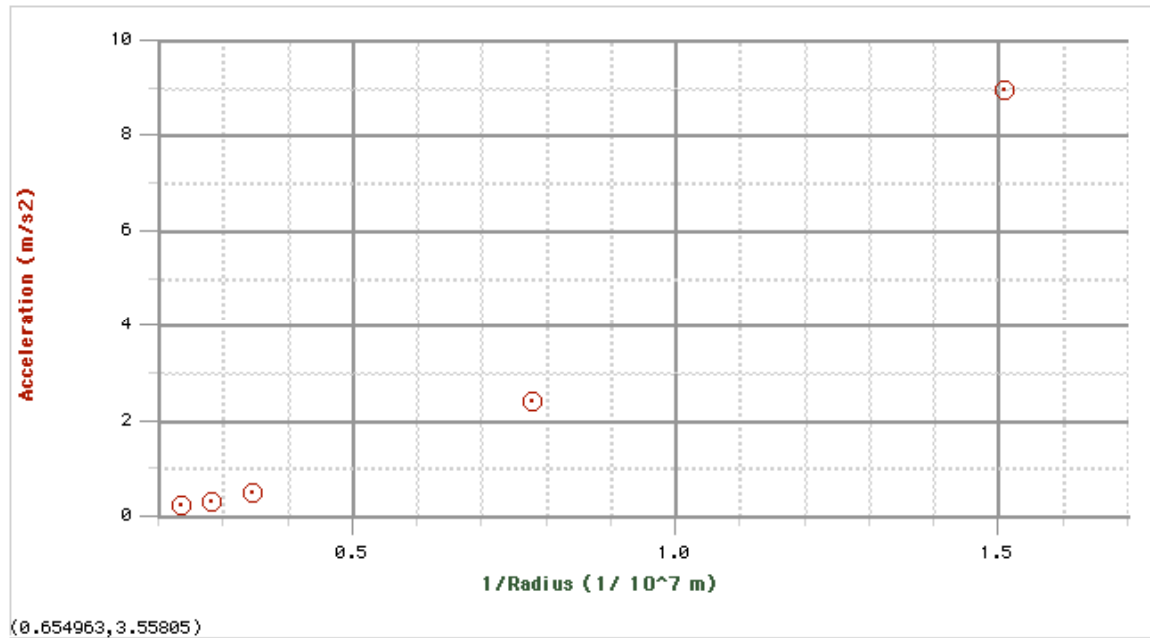
Verbal Model: As the radius of the orbit increases, the acceleration of the satellite decreases.

Math Model: $a = k (1/r)$

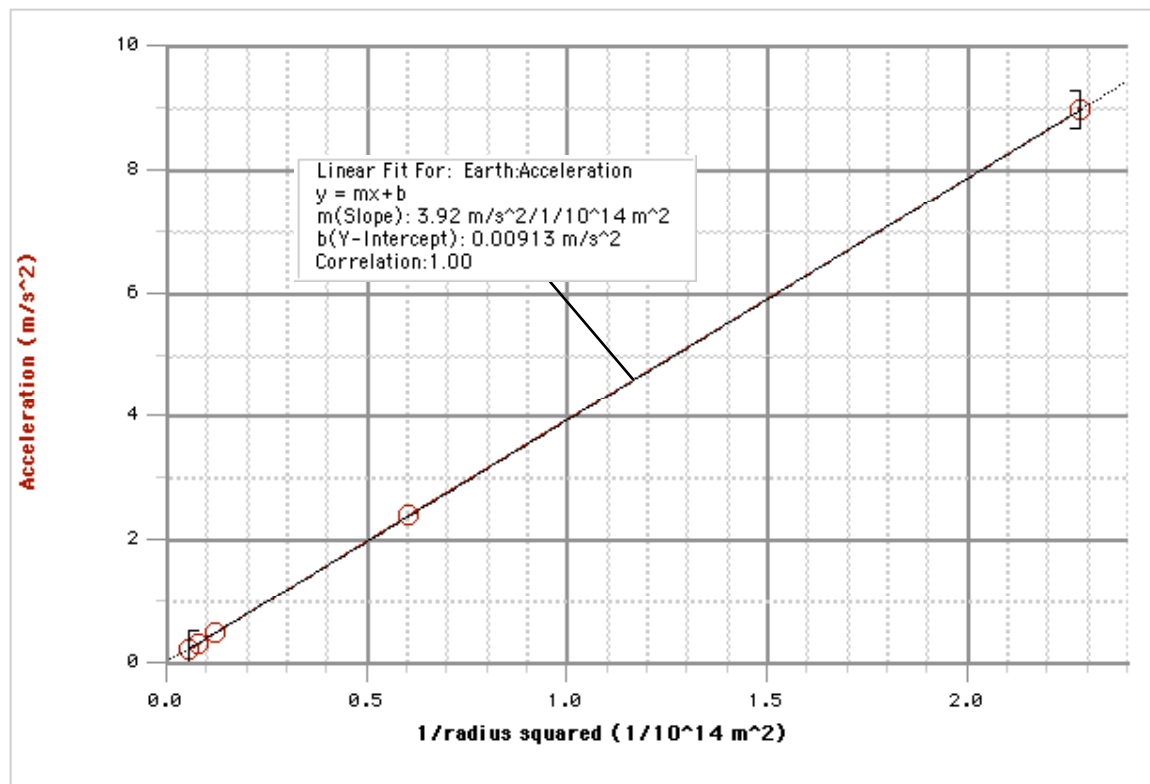
Since a curved graph does not give any direct information about the precise mathematical equation describing this relationship, it is necessary to linearize the graph to get this valuable information (a common approach in the Modeling Method). *Graphical Analysis* does have a curve fitting feature, but it is best not to use this tool. Students should produce test plots of different relationships until they find the one that shows the graph as linear. Students have an easier time analyzing a linear relationship, rather than the equation that seems to magically appear when done by the computer curve fitting tool.

In order to linearize the graph, students will most likely first try to modify the graph by plotting a vs. $1/r$ since the graph is an inverse relationship. In order to produce the test plot, students should create a new column of data labeled " $1/r$ ". They can create this column by hand, or *Graphical Analysis* can do it for them by going to the "Data" menu and selecting "New Calculated Column". You can then enter the formula you wish the computer to use to create a new data column. Remember to also change the units of the column.

This however, is not the correct relationship, and so when the students try this test plot, they will see that the graph is still not linear.



Thus they should try another type of inverse, a vs. $1/r^2$. It is likely that students have not yet seen an inverse squared relationship, and so they may need a hint at this point. When they plot a vs. $1/r^2$, they should see that their data now fit a straight line. On *Graphical Analysis*, use either the "Linear Fit" function or "Regression" to have the computer fit a line to the data, and give values for slope and y-intercept.



Verbal Model: As the inverse square of the radius increases, the acceleration of the satellite increases proportionally.

Math Model: $a = (3.92 \cdot 10^{14} \text{ m}^3/\text{s}^2) (1/r^2) + 0 \text{ m/s}^2$

It is not necessary at this point to explain the meanings of the slope and y-intercept.

Post Lab Discussion

After all data has been collected and graphs have been drawn and analyzed, each group should write their findings on their whiteboard. Possible things to draw on the whiteboard are: the a vs. $1/r$ and a vs. $1/r^2$ graphs and their explanations, the process of finding the acceleration, the data table, the parent body used, and the purpose of the lab. The teacher can decide how much detail to include.

After the whiteboards are drawn, the groups should display them in the front of the room so all can see them. The teacher should call on one group at a time to present their findings and answer questions from the class. The teacher should first focus on making sure the graphs and explanations are correct. If not, ask the class to help the group modify their explanations or figure out what went wrong in their process. A few groups (with different parent bodies) should present, but it is not necessary for every group to do so.

As students whiteboard, the teacher should have the students find similarities and differences between each group's results. The major similarity should be that the general shapes of the graphs are the same, therefore showing that the relationship between acceleration and radius is the same no matter what parent body or satellites are chosen. Make sure that students understand the idea that acceleration is proportional to the inverse square of the radius. For example, ask what happens if you move the satellite twice as far from the center of the parent body? The acceleration is $1/4$ of the original value.

It is easier to understand this phenomenon by thinking in terms of forces rather than accelerations. Ask students to remember Newton's 2nd Law, where $\Sigma F = ma$. Therefore we can say that the acceleration is proportional to the force, so that as the acceleration increases, the force increases as well. Relating to this lab, when the radius increases, the acceleration decreases, so according to Newton's 2nd Law, the forces decreases as well. We can then draw the conclusion that the force of gravity on the satellite is less at greater distances from the parent body.

In terms of the motion they observe, we need to explain why the satellite moves slower at farther distances. If we know that $a = v^2/r$ and we also know from our graph that $a \propto 1/r^2$, then we can conclude that $v^2/r \propto 1/r^2$. Canceling one of the radius terms, we find that

$$v^2 \propto 1/r$$

This explains why the satellite moves slower when it is farther away from the parent body, as seen in the original simulation.

Add RICH's explanation Re: projectile around earth XXXXXXXXXXXXXXXX

The major difference that students should note is that the slopes are very different for different parent bodies. In this lab, it is not very obvious what the slope of this line represents, as the units (m^3/s^2) do not provide much clue as to the physical meaning of the slope. Since we cannot easily find the meaning of the slope, ask students what is different about the parent bodies that may account for the difference in slope. Hopefully, they will suggest the mass. Since the mass is different for each parent body, and the slope of the graph is different for each, one can make an educated guess that the slope has something to do with the parent body mass (larger masses have steeper slopes). The slope itself is not the mass, since the units do not match, but the slope is somehow related to the mass. This idea will be addressed further in the next lab (Lab Activity 3).

From the a vs. $1/r^2$ graph, we find the following equation:

$$a = k (1/r^2)$$

where k is the slope of the graph.

Therefore we can say that the acceleration is proportional to the inverse square of the radius:

$$a \propto 1/r^2$$

Or that when the radius increases, the acceleration decreases.

Finally, according to Newton's 2nd Law, force is proportional to acceleration so we can rewrite this proportionality as

$$F \propto 1/r^2$$

Or, in words, the force is larger when the satellite is closer to the parent body.

Unit VIIIa

Laboratory Activity 3: Cavendish's Experiment and the Determination of G

Teacher Notes

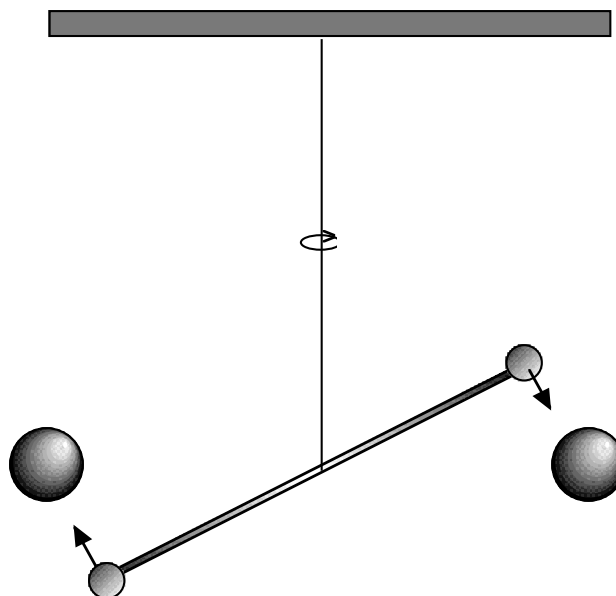
Newton demonstrated that gravitational forces are dependent upon the masses of the object involved and the distance between their centers. These relationships can be represented mathematically by

$$F_g \propto \frac{m_1 m_2}{r^2}$$

In order to make this proportionality an equivalency, a constant needs to be introduced such that appropriate units of mass and distance will yield a calculated force in Newtons. The full version of what is now known as Newton's Universal Law of Gravitation can be expressed as

$$F_g = G \frac{m_1 m_2}{r^2}$$

Where G represents the Gravitation Constant. In Newton's day, the value of G was not known. To determine the value of G from the Universal Law would require being able to measure the force of gravity between two known masses separated by a known distance. The gravitational force between two masses that would practically fit in a laboratory would be exceptionally small and therefore extremely difficult to measure. An effective method of measuring such small forces was not developed for over one hundred years after Newton's initial work on the Universal Law.



The process of measuring such small forces involves the use of a torsion balance, represented schematically at left. The spheres would usually be made of a dense material such as lead or some other metal. When the larger spheres are placed close to the smaller spheres supported by the thread, the thread will twist slightly due to the gravitational attraction between the metallic spheres. The gravitational forces could be quantified if the thread was first subjected to very small known forces and the resulting twisting measured carefully. Since the masses of the spheres and the distances between them can be easily measured, the value of G can be calculated.

A Sample Calculation

Suppose the larger lead spheres each have a mass of 10 Kg. The smaller lead spheres have a mass of 2 Kg. The distance between the centers of these spheres is set at 0.1 meter. Suppose also that the torsion balance measures a force of attraction between the spheres of 1.33×10^{-6} Newtons. Use these values to determine the value of G from Newton's Universal Law of Gravitation.

Cavendish obtained a value slightly larger than that calculated from the example above. Amazingly, his value was remarkably close to the currently accepted value. This was quite an accomplishment considering the extremely small forces involved and the numerous potential sources of experimental error. Even the slightest air currents or vibrations would make accurate readings impossible. The currently accepted value of G is

$$6.67 \times 10^{-11} \frac{N \cdot m^2}{Kg^2}$$

The units can be confusing, but it should be clear that the units are such that when the value of G is used in the Universal Law of Gravitation, the end result is a calculated force in Newtons.

The significance of Cavendish's contribution to physics and astronomy cannot be overstated. With the value of G, the mass of the Earth, and, in fact, of any other body for which appropriate orbital data can be measured, can be calculated. In short, determining the value of G opened up the possibility of weighing the cosmos.

PART 2: Finding the Constant of Universal Gravitation, G, from the Lab Activity 2 data

***NOTE HERE ABOUT THE CIRCULAR REASONING OF THIS....XXXXXXXXXX..

Ask students to remember back to Lab 2, where the slopes of the a vs. $1/r^2$ graphs were different for different parent bodies. We assumed that there was some relationship between the slope of the graph and the mass of the parent body. It is now possible to confirm this constant of proportionality, which is G, the value determined by Cavendish. In the last lab, the equation we found was

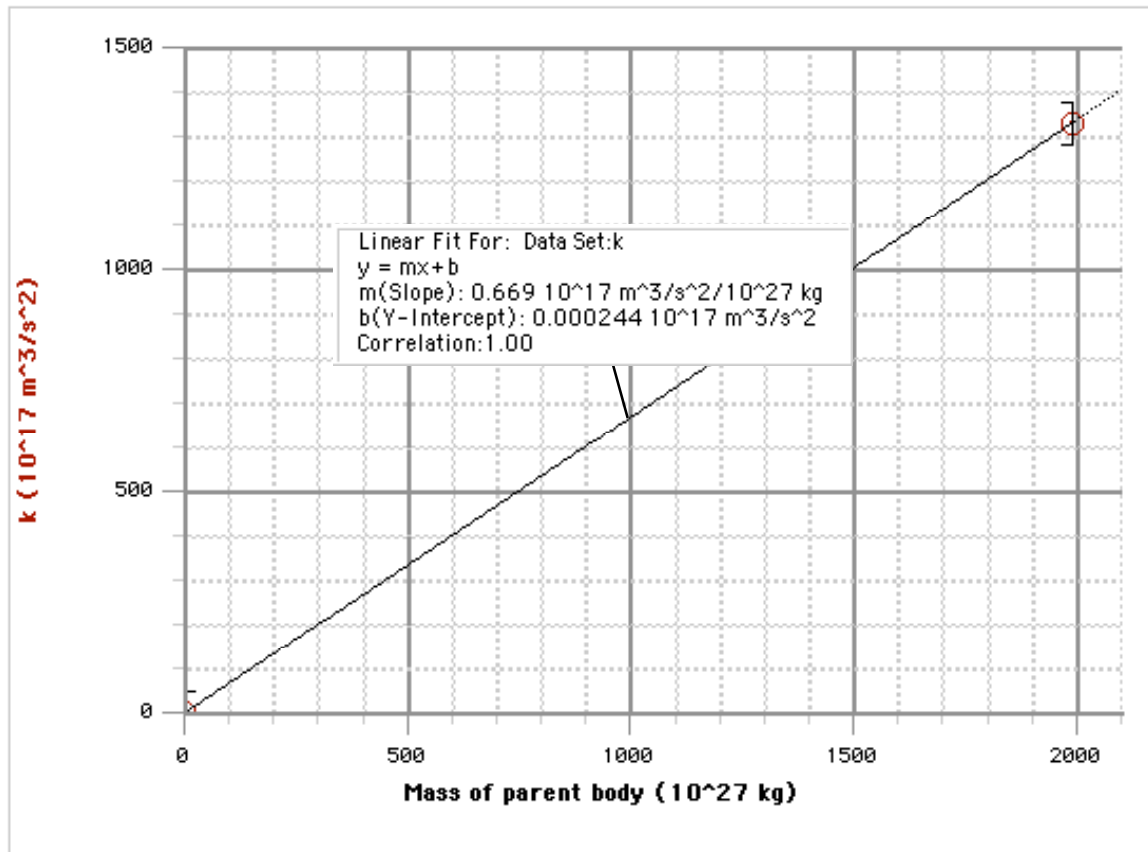
$$a = k(1/r^2)$$

Students can now share data between groups to make a new data table including the mass of the parent body and the slope of the a vs. $1/r^2$ graph. The masses of the parent bodies will need to be looked up in a table.

Parent Body	Mass (kg)	Slope of line (k) (m^3/s^2)
Earth	0.0059742×10^{27}	0.00392×10^{17}
Uranus	0.08722×10^{27}	0.0578×10^{17}
Sun	1989.1×10^{27}	$1330. \times 10^{17}$

Students can then graph this relationship, making sure to put all numbers of the same variable to the same power of 10. When graphing, again enter only the mantissa in the data column and enter the exponent in with the units.

This should produce a linear graph. Graphical Analysis will then calculate the slope, and the students will have to modify the slope number by including the exponents shown with the units.



The slope should come out very close to $6.67 \cdot 10^{-11} \text{ m}^3/\text{s}^2\text{kg}$, which is the Constant of Universal Gravitation, G . The more common units are Nm^2/kg^2 , and you may want to have the students show how these units are equivalent.

Working backward, if we know that the slope of the line is G (comparing to the value determined by Cavendish), then the equation for this line is

$$k = GM$$

and therefore substituting this in our original equation we get

$$a = GM(1/r^2)$$

Again, if we want to speak in terms of forces rather than accelerations, we can multiply the acceleration by the mass of the satellite (m) to get the force on the satellite. We will need to multiply the other side of the equation by the mass of the satellite as well.

$$ma = m GM(1/r^2)$$

$$F = GMm/r^2$$

Since the "a" in the original equation was the centripetal acceleration, the F must be the centripetal force, F_c . We can rewrite the equation as

$$F_c = GMm/r^2$$

Thinking back to the original reasoning for Lab 2, we said that the only force acting on the satellite was the force of gravity. The force of gravity, therefore, must be the centripetal force.

$$F_c = F_g$$

And therefore,

$$F_g = GMm/r^2$$

which is Newton's Law of Universal Gravitation.

Name _____ Date _____ Per. _____

Unit VIIIa

Worksheet 2

The National Academy of Sciences, in order to gather information in deforestation, wishes to place a 500 kg infrared-sensing satellite in a polar orbit around the Earth. The radius of the Earth is approximately 6.38×10^3 km, and the acceleration of gravity at the orbital altitude of 160 km is very nearly the same as it is at the surface of the Earth. Show all work.

1. Construct a force diagram for the satellite described in the above statement.

2. What is the agent of the centripetal force for the satellite?

3. How much work is done on the satellite during one complete orbit of the Earth?
Explain your answer.

4. Determine how long it will take for the satellite to make one complete revolution around the Earth. (From acceleration data determine the average circular velocity.)

The Earth's orbit around the sun is very nearly circular, with an average radius of 1.5×10^8 km or 1.5×10^{11} m. Assume the mass of the Earth is 6×10^{24} kg and the mass of the Sun is 2×10^{30} kg. One year is equal to 3.156×10^7 seconds. $G = 6.672 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

5. What is the average speed (km/sec and m/s) of the Earth in its orbit around the Sun?

- 6. What is the magnitude of the Earth's average acceleration in its orbit around the Sun?**
- 7. With how much force does the Sun attract the Earth?**
- 8. With how much force does the Earth attract the Sun?**
- 9. Using Newton's Law of Universal Gravitation ($F = Gm_1m_2/r^2$), what happens to the Force when you double m_1 ? When you double m_2 ?**
- 10. Using Newton's Law again, what happens to the Force when you double r (the distance between the centers of the 2 masses)? When you triple r ?**
- 11. Calculate the gravitational force between the Sun and the Earth. How does this compare with the answer to question 7?**
- 12. Calculate the gravitational force between the Moon and the Earth (Moon mass = 7.35×10^{22} kg with a distance between the mass centers of 384,000 km). Don't forget to convert to m.**

Unit VIIIa

Derivation of Kepler's Third Law From Newton's Laws of Motion & Law of Universal Gravitation

Teacher Notes

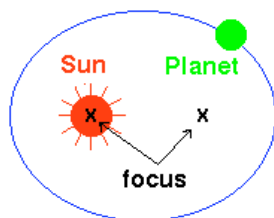
Newton believed that the influence of the Sun forced the natural straight-line motion of the planet into a curve. He demonstrated Kepler's Laws would be true if and only if forces exerted on the planets are always directed toward a single point. Such a force is called a central force. Planets obey the law of areas (Kepler's Second Law) no matter what the magnitude of the force, as long as the force is directed toward the same point (i.e., the Sun). How then do we demonstrate that a central gravitational force would cause the exact relationship observed between the orbital radii and the period of the planets, as given by Kepler's Third Law?¹

Newton's Laws of Motion:

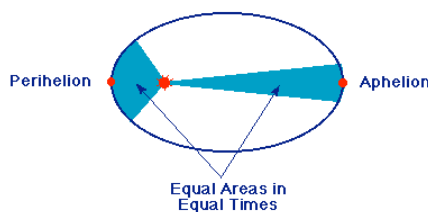
1. Every object continues in its original state (at rest or in uniform motion – uniform speed in a straight line) unless acted upon by an unbalanced (net) force.
 - a. If an object is at rest or in uniform motion, then the forces acting on it must sum to zero (a zero net force).
2. The change in motion of an object (acceleration) is equal to the ratio of the net force acting on the object and the object's mass: $a = F/m$. This equation also shows then that the net force is equal numerically to and in the same direction as the acceleration of the object multiplied by its mass: $F_{\text{net}} = ma$.
3. If one object exerts a force on a second object, the second object in turn exerts an equal but oppositely directed force on the first object.

Kepler's Three Laws of Planetary Motion:

1. The planets orbit the Sun in ellipses with the Sun at one focus and nothing at the other focus.



2. An imaginary line connecting the Sun and a planet sweeps out equal areas in equal time



3. The squares of the periods of the planets are proportional to the cubes of the average distances from the Sun.
(Law of Periods)

$$\frac{P_1^2}{P_2^2} = \frac{R_1^3}{R_2^3}$$

¹Cassidy, Holton, Rutherford, *Understanding Physics*, p.179, Springer-Verlag, New York 2002

Newton observed that elliptical orbits of the planets around the Sun had to be the effect due to a central force directed at the Sun. Using his Second Law and a geometrical analysis of planet motion, Newton showed that planets move according to Kepler's Law of Areas. He then found that motion in an elliptical path would only occur when the central force was an inverse-square force: $F_c \propto 1/R^2$. Thus, only an inverse square distance force exerted by the Sun would cause the elliptical orbits of planets observed by Kepler. Newton then proved the argument by showing how this inverse square law would result in Kepler's Third Law, $T^2 \propto R^3$:

Let's consider the special case of an ellipse that is a circle.

Since the centripetal force equals the force of gravity for an orbiting body

$$F_c = F_g$$

$$ma_c = \frac{GMm}{r^2}$$

The formula for centripetal acceleration a_c of a body moving in a circular path with radius R and period T :

$$a_c = \frac{v^2}{R} \quad \text{and velocity is: } v = \frac{2\pi R}{T}$$

Therefore, combining the above expressions yields

$$a_c = \frac{4\pi^2 R^2}{T^2} \cdot \frac{1}{R} = \frac{4\pi^2 R}{T^2}$$

Substituting this expression for a_c into the original force equation gives

$$\frac{m \cdot 4\pi^2 R}{T^2} = \frac{GmM_{\text{sun}}}{R^2}$$

Dividing out mass of the planet and using algebra, Newton showed Kepler's Law of Periods:

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM_{\text{sun}}} = \text{constant}$$

(If the units of R are AU (astronomical units) and the units of time are Earth years, then $R^3 = T^2$.)

Newton still had more evidence from telescopic observations of Jupiter's moons and Saturn's moons. These moons also obeyed Kepler's Laws with each planet as the larger, central mass:

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM_{\text{planet}}} = \text{constant}$$

with a different constant for each planet (from reviewing the equation, the difference is due to the different masses of the central object).

Using this combined Kepler-Newton equation, one can now determine the mass (kg) of the central object (assuming the mass of the moon or satellite is much smaller than the central object mass and assuming circular orbits) by just knowing the orbital period (T in sec) of the satellite and its orbital radius (R in m as measured from the centers of the objects):

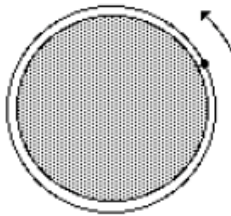
$$M = \frac{(4\pi^2) (R^3)}{G T^2}$$

Where $G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

Unit VIIIa: Worksheet 3

1. The gravitational field strength on the Moon, which has a radius of 1.74×10^6 m, is approximately 0.17 as large as the gravitational field strength at the surface of the Earth. How much would a 1500 kg satellite weigh at the surface of the Moon?

Assume the diagram below represents the orbit of the satellite around the Moon at an altitude of 100 km.



2. Construct a force diagram of the satellite in orbit. What is the direction of the acceleration of the satellite?
3. What is the orbital radius of the satellite?
4. What is the orbital speed of the satellite?
5. What is the orbital period (in hours)?
6. If the satellite were to change its orbit so that it was now at an altitude of 50 km, would its velocity have to increase or decrease to remain in orbit? By what factor would the velocity change? Explain.

Name _____ Date _____ Per. _____

Unit VIIIa

Worksheet 4

Orbital Data for the Planets

Quantity	Units	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Distance from Sun										
Semimajor axis (R)	AU	0.39	0.72	1.00	1.52	5.20	9.55	19.22	30.11	39.44
	10 ⁶ km	57.9	108.2	149.6	227.9	778.3	1429.4	2875.0	4504.3	5900.1
Minimum (perihelion)	AU	0.39	0.72	1.00	1.52	5.20	9.55	19.22	30.11	39.44
	10 ⁶ km	45.9	107.4	147.1	206.7	740.9	1347	2735	4456	4425
Maximum (aphelion)	AU	0.47	0.73	1.02	1.67	5.45	10.07	20.08	30.33	49.30
	10 ⁶ km	69.7	109.0	152.1	249.1	815.7	1507	3004	4537	7375
Orbital period										
Sidereal (T)	years	0.241	0.615	1.00	1.88	11.86	29.46	84.01	164.79	247.69
Synodic	days	115.9	583.9	-	779.9	398.9	378.1	369.7	367.5	366.7
Inclination of orbit to ecliptic	degrees	7.00	3.39	0.0	1.85	1.30	2.49	0.77	1.77	17.2
Eccentricity	-	0.206	0.007	0.017	0.093	0.049	0.056	0.047	0.009	0.250
Mean orbital velocity	km/s	47.89	35.03	9.79	24.13	13.06	9.64	6.81	5.43	4.74

1. Applying Kepler's Law of Periods to the first 6 planets, determine the average value of the constant R^3 / T^2 using AU and Earth years.
2. Using meters and seconds for the orbital elements of the first 6 planets, determine the average value for the Mass of the Sun. (1 year = 3.156×10^7 s)

Unit VIIIa

Laboratory Activity 4: Geometry of the Jupiter-Moons System

Teacher Notes

Objectives

- 1) That students can measure period and orbital radius of a moon going around a planet when viewed from the side.
- 2) That students understand that these are the two variables needed to determine the mass of the central planet using Kepler's 3rd Law.

Pre-lab discussion

Show the Quicktime movie, JupitersMoons.mov. This was created using the software "Starry Night Pro". Detailed instructions on how to prepare your own are included in the separate "Teacher's Notes".

Ask students for observations of what they're viewing. They will probably say things about the speeds of the moons, period of orbits, tilt of orbits, shape or size of orbits, rotation of Jupiter, mass, etc. What do they notice about the apparent speed of the moon at different positions in the orbit? What about the speeds of the moons relative to each other? What are the variables involved? Lead them toward observations about period and orbital radius if they don't naturally emerge.

Assignment in student groups

Part I

Make a sketch of what the Jupiter-Moons orbits would look like if we could view them from the "top", i.e. directly above Jupiter's axis of rotation. Make 2 more sketches that show how they would appear as our view shifts away from the axis. Now make a sketch of this system when viewed from the side or edge, i.e. 90 degrees off axis. What view do you think we are seeing in the movie shown above? (Teacher: You might like to have this movie playing in "loop" mode somewhere in the room during this activity.) Make one final sketch showing how one moon would appear at 6 one-hour intervals, when viewed from the side. Assume its period is much larger than 6 hours. Prepare your whiteboard presentation summarizing your results.

The expectation here would be rough sketches rather than artistic drawings. Look for orbital shapes that vary from being circles (top view) to ellipses (view at an angle) to straight lines (side view). The final sketch of the single moon would be 6 points in a straight line perpendicular to Jupiter's rotational axis. They may or may not be uniformly spaced. Accept either one at this point and allow for some discussion during the whiteboard presentations.

Whiteboard presentations

Students do whiteboard presentations of the work in Part I before going on to Part II. It is particularly important to focus the students on the edge view, since that is the one we actually have of Jupiter from the earth and will use later on. During this time the following concepts should be introduced and defined: orbital radius, period, turnaround points (the extreme horizontal positions of the moons), transit region (when the moon passes in front of the planet) and occultation region (when the moon passes behind the planet).

End this time by asking the students the following two questions: 1) “If you have only the side view of the moon’s orbit, at what position or positions can you measure its orbital radius?” 2) “How would you measure the period of this same moon?” This could be a short group discussion. Hopefully the students will see: 1) that the orbital radius can only be measured at the two turnaround points; 2) that the period is the time for the moon to return to the same starting spot moving in the same direction.

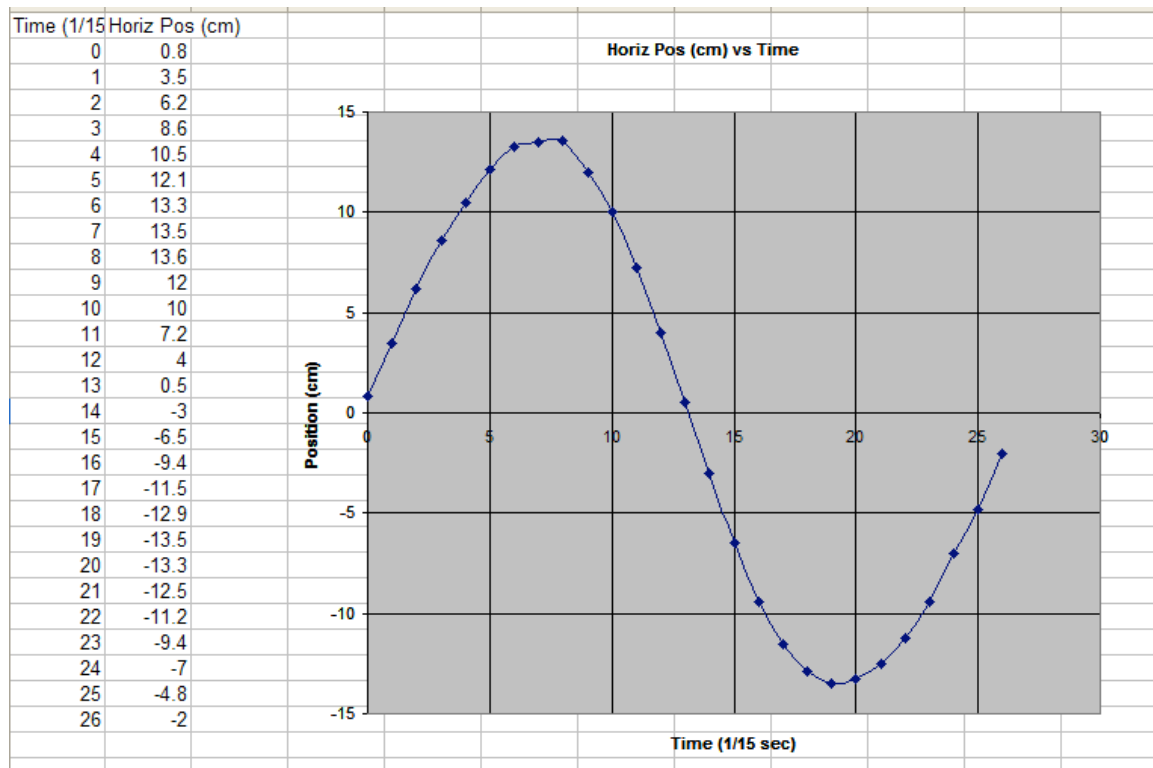
Part II

Create a physical model that simulates the rotation of one moon around a planet when viewed from the edge. The model must allow you to take data of position of the moon versus time. Take data from your model for at least one orbit of the moon around the planet. Create an appropriate presentation of this data. Determine the orbital radius and period of the moon. If you had this particular data of an actual moon around an actual planet, how would you use it to determine the mass of the planet?

Note to teacher: This activity can be left completely open-ended, or students can be given access to a turntable with a digital or video camera, for example. Digital cameras which do not have a Quicktime movie mode can be used to record still images of the “moon” shown in multiple positions. There are many other possibilities for models. 1) Students can suspend a mass from a string, let it hang vertically and move it in a circle. The view from the side is identical to the turntable. A camera could capture this view. 2) Students can draw a circle for an orbit on a piece of paper and locate small balls or marbles at uniform positions around the circle. They can then either sight or physically measure distances on this setup. 3) #2 above could be done without using balls and simply marking uniform positions along the circle. Measurements can be taken with rulers. (For details of this method see “Hands-On Universe” curriculum booklet, Measuring Size, Supplementary Activity 3: Simulating Orbits (pp. 20-21), and Teacher Notes, pp. 32-34).

Included in this curriculum guide is another Quicktime movie (Turntable Model3.mov) from which students may take position and time data. In the movie, positions to the left of center are considered to be negative. The “moon” may be moved in increments of 1/15 sec. by hitting the right arrow key on the computer. The scale in the background shows units of 1 cm. An explanation of the creation of this movie is given in the “Teachers Notes”. An Excel spreadsheet of data taken from this movie is given in the file, TurntableSim3.xls, shown below. The software, “Graphical Analysis”, can easily be

used instead, taking advantage of its plotting and curve fitting features. The goal of this process will be to determine the period (T) and the orbital radius (r) of the moon.



The orbital radius is easily found from the above plot by measuring the “amplitude” of the apparent sine wave. This value is approximately 13 cm.

The period of the orbit can be obtained in a number of different ways. For example, by extrapolating the left and right ends of the graph to the zero position the full period is approximately 27.5 time units. Each time unit is 1/15 sec, so the period is 1.8 seconds. Since this turntable was rotating at 33 1/3 rpm, ideally, 1.8 seconds is a reasonable result.

Whiteboard presentations

Students present their data to the entire class, explaining the model they used, their methods for taking and analyzing data, and their result of orbital radius and period.

Supplemental Notes for Laboratory Activity 4 “Geometry of the Jupiter-Moons System”

Creating the Jupiter’s Moons movie from the “Starry Night Pro” software:

- 1) Open the program. At the upper left, below the tools, make sure “Time”, “Planets” and “Display” are turned ON.
- 2) Turn off “Daylight” and “Horizon” using the pull-down Sky menu at the top or the menu at the left side. Also on the left side turn off “Stars” and make sure “Planets” are turned on. You may want to turn other celestial objects OFF as well, but they

- don't make too much difference in this case. You should have a dark sky at this point.
- 3) Go to "Location" and set "Fixed heliocentric position". Click on "Set Location".
 - 4) Go to "Planets" window on lower right and double-click on Jupiter. This will center Jupiter in the dark screen and lock it there.
 - 5) Depending on the time of the display it is possible that you may have a planet, or the moon, in your display or covering up Jupiter. If that is the case, run the time forward or backward to move these objects out of the way. This is done in the "Time" window at the lower right and pressing the appropriate arrow.
 - 6) Zoom in on Jupiter by pressing the + magnifier about half-way down the left-hand menu. Keep pressing this button until the moons of Jupiter appear and spread away from the planet. Jupiter also changes from a white dot to an actual image of the planet.
 - 7) In the "Time" window set the minutes to 20 (each frame represents an actual 20 minutes) and run the simulation. You will see the moons moving in their orbits and Jupiter rotating.
 - 8) To shift the display so that the moons are moving horizontally, go to the tools in the upper right corner. Press the rotation tool, just below the hand, and apply this to the display as needed for the orientation you desire.
 - 9) At this point you may need to adjust the magnification so that the outermost orbit extends over the entire screen.
 - 10) This simulation may now be run as long as you want for viewing by the students. If, however, you wish to create a short, Quicktime movie, you will use the movie icon (tool in upper right corner) and consult the "Starry Night Pro" User's Manual for instructions on "Making Movies". If you don't have a hard copy of the manual, it is available online through the Help menu. The advantage of this step is that the movie you create can be played on any computer which has Quicktime. It doesn't require Starry Night Pro.

Creating the Turntable Model movie and recording the data:

This simple, physical model uses an old record player turntable with a ping pong ball taped to the outer edge of the turntable. Black paper is used to cover up as much of the changer mechanism as possible. A scale in 1 cm units is placed behind the turntable. The zero point on the scale is adjusted to coincide with the central axis of the turntable. The turntable is set to rotate at the slowest speed possible, in this case at $33 \frac{1}{3}$ rpm.

An Olympus digital camera (D-510) with a Quicktime movie feature was set on a tripod in front of the apparatus and at the same level of the turntable. This allowed a side view of the rotation. The zoom was set so that the turntable filled the screen and the ball could be seen at both horizontal extremes. This particular camera runs a standard 15 second sequence of frames at 1/15 sec/frame. The file is then downloaded to a computer and run with Quicktime.

To record data the movie is incremented, frame by frame, using the right arrow key. At each position the distance of the ball from the axis center is measured. Time increments are at 1/15 sec. The data table is recorded and processed in Excel, Graphical Analysis, etc.

Unit VIIIa

Laboratory Activity 5: Mass of Jupiter Lab

Teacher Notes

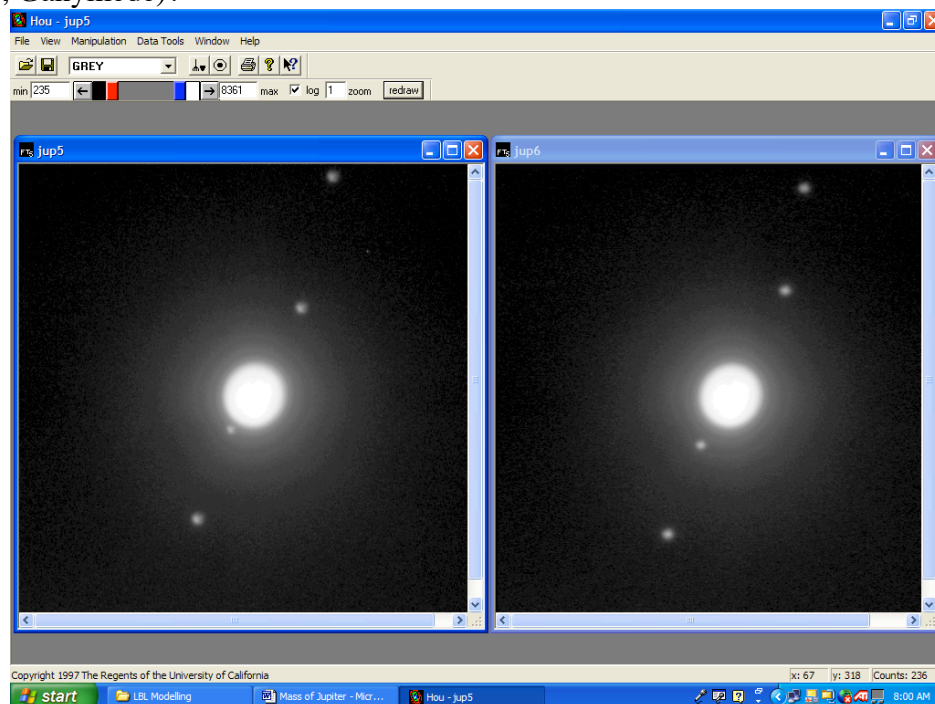
In order for this lab to be done, students and teachers must have a working knowledge of the “Hands-On Universe©” Image Processing software. See <http://lhs.berkeley.edu/hou>

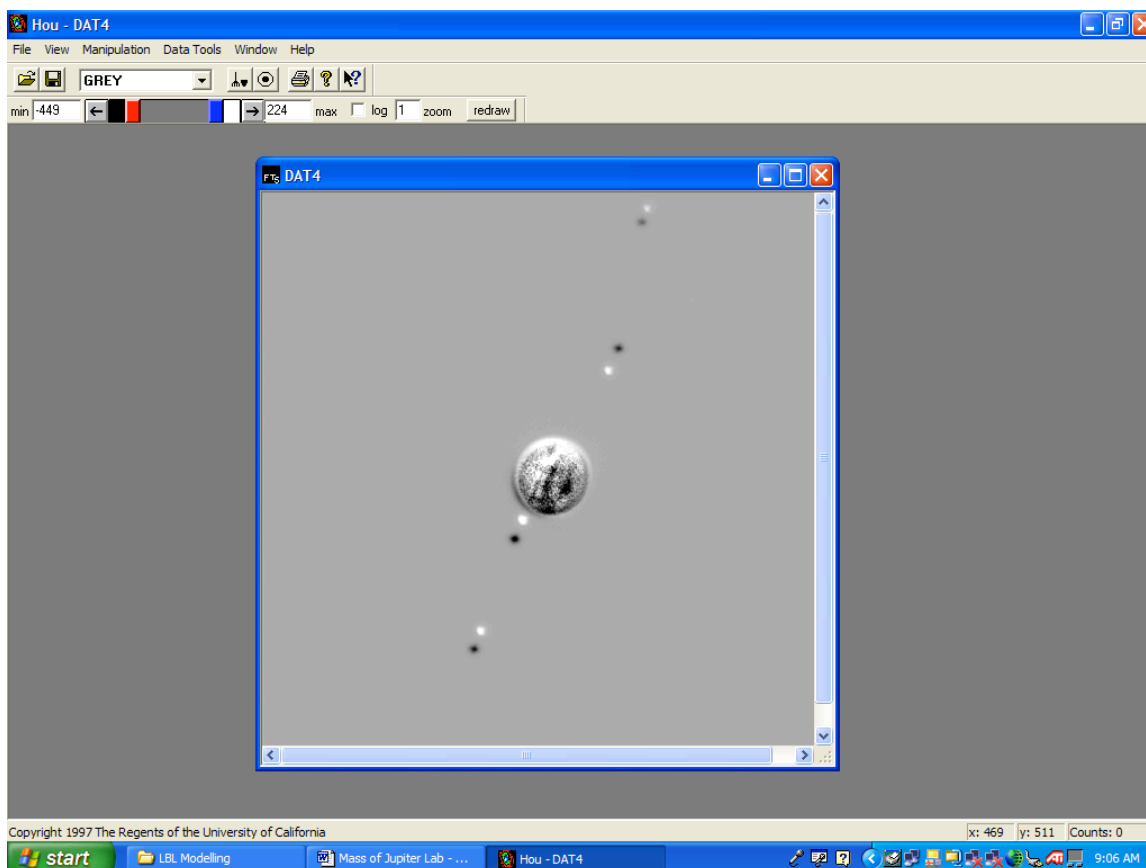
Objective

The students will take data from a set of real images of Jupiter and its moons to determine the mass of Jupiter. This will require finding a way to measure the orbital radius and period of one of the moons.

Pre-lab discussion

Using the “Hands-On Universe©” Image Processing software display the images, Jup5 and Jup6, side by side on the computer screen. These are from the unit, Tracking Jupiter’s Moons. (See screen shot below.) Tell the students that these are actual images taken one hour apart. What do they see? Encourage observations about position, motion, direction, etc. of the moons. Can they easily tell relative position between the two images? Go through the procedure of subtracting Jup6 from Jup5 to display the data in a single image. This would be a good review of this procedure. (See the second screen shot below. The white dots are from Jup5 and the black dots from Jup6.) Now what do they see? The position and direction of motion will be much clearer. Which moon appears to moving fastest? Slowest? Any idea of which moon is which (Io, Europa, Callisto, Ganymede)?





Student Lab

This lab is already included in the “Hands-On Universe©” curriculum booklet, Measuring Size. (Tracking Jupiter’s Moons Unit, pp. 13-19, and The Mass of Jupiter Unit, pp. 56-61) Teacher notes are included in the “HOU” Teacher Book (pp. 40-42). It is not necessary for the students to complete all activities in these two units. For example, if students are already familiar with the “HOU” image processing software tools, including the procedure for adding and/or subtracting images, they can easily begin with Activity III (“What Happens to the Moons During Six Hours?”). They would follow steps 5 through 7, only. Then they move to The Mass of Jupiter Unit.

To do this activity requires use of computers for viewing the images. Working in pairs or triads around a single computer is often a very effective way to involve students in collaboration and discussion. If sufficient computers are not available then printouts of the individual images and/or the composite image can be used. In that case the working groups could be larger.

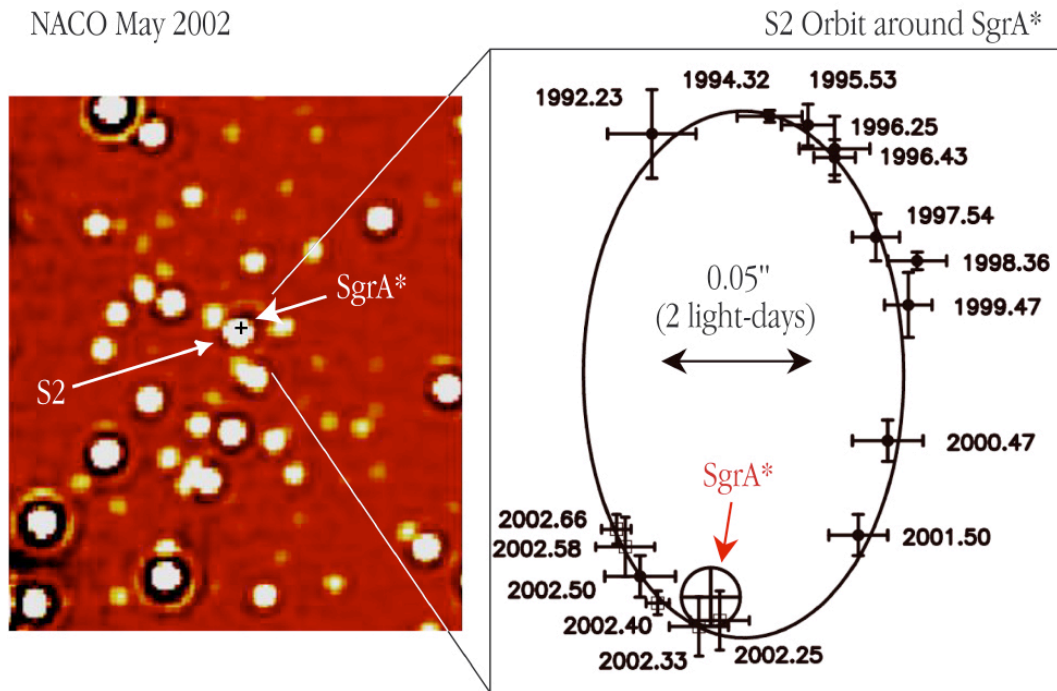
The goal of this lab is, ultimately, to calculate the mass of Jupiter by measuring the orbital radius and period of one of Jupiter’s moons. Kepler’s 3rd Law is then used for the final calculation of mass. The difficulty for the students will be in dealing with data that represents only a partial period of an orbit. It is recommended that the emphasis of this

activity be on the method or methods used to determine radius and period, and not on the final mass calculation. Encourage a variety of methods throughout the class.

Post-lab Discussion during Whiteboarding

Students should include in their presentations a discussion of the following: 1) the method for finding the orbital radius and period of one moon and their rationale for choosing that particular moon; 2) the final calculation of the mass of Jupiter and its comparison to the published value; 3) the sources of error in their method, given that they had to use incomplete data. A rich source of dialogue after the presentations are completed can be a group debate over which method is “the most accurate”. The students may be able to come to the conclusion that the method that yields the closest value to the published mass is not necessarily the most accurate method. The mass calculation depends upon both radius and period, and major errors in both numbers can, possibly, cancel each other. (Recall: $M = 4\pi^2 r^3 / GT^2$)

Worksheet 5: Mass of Black Hole at the Center of the Milky Way Galaxy



The Motion of a Star around the Central Black Hole in the Milky Way

ESO PR Photo 23c/02 (9 October 2002)

© European Southern Observatory



"SgrA*" and S2 are identified in the left panel. The right panel displays the orbit of S2 as observed between 1992 and 2002, relative to SgrA* (marked with a circle). The positions of S2 at the different epochs are indicated by crosses with the dates (expressed in fractions of the year) shown at each point. The size of the crosses indicates the measurement errors. The solid curve is the best-fitting elliptical orbit - one of the foci is at the position of SgrA*. Credit: ESO

- Using the Kepler-Newton equation, $M = (4\pi^2 / G) (R^3 / T^2)$, *determine the Mass of the central black hole* using the orbital data from the above graph. You will note that R is replaced by a, the semi-major axis of S2 elliptical orbit. $G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
 - The orbital period is 15.2 years (reference: 1 year = 3.16×10^7 sec)
 - The closest point of S2 to SgrA* is 17 light-hours; the furthest point of S2 from SgrA* is 10 light-days (reference: 1 light-hour = 1.08×10^{12} m)
 - The semi-major axis (a) is equal to the greatest distance (the line through the foci of the ellipse) across the ellipse. Use this value 'a' for R.
- Another group used the same data points and determined the orbital elements of S2 were a period of 15.6 days with a periastron (closest point) of 17 light-hours and an apastron (furthest point) of 5 light-days. *Calculate the mass of the central black hole* with this data

Unit VIIIa

WS5 Teacher Notes: Applying Kepler's Third Law to Non-Circular Elliptical Orbits

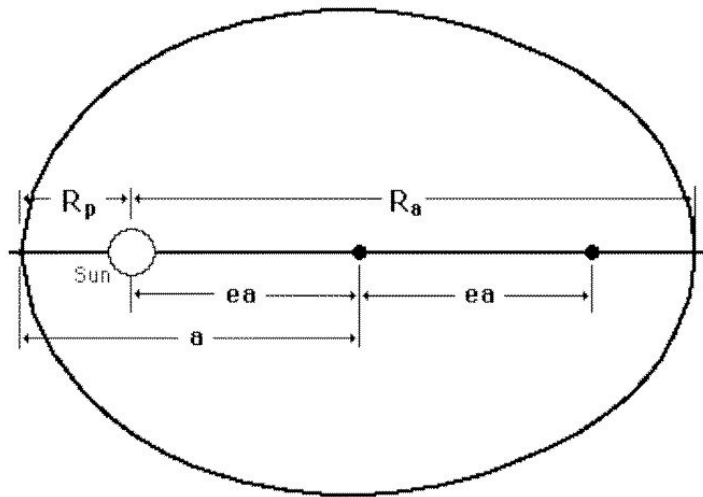
The only difference is to use 'a' (semi-major axis) in place of R in the Kepler-Newton Mass Equation.

Reference: 1 year = 3.16×10^7 sec; 1 light-hour = 1.08×10^{12} m

Kepler's Laws of Motion can be applied universally to all kinds of orbiting bodies in space, including stars around massive black holes.

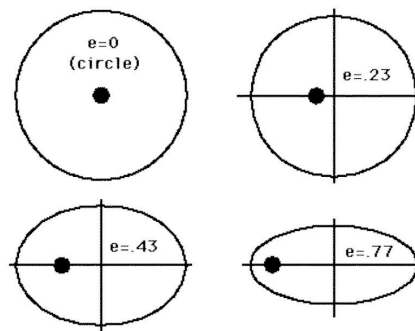
- a = semimajor axis
- e = eccentricity
- R_a = Aphelion radius
- R_p = Perihelion radius

The orbital point that is farthest from the central body is called the aphelion (Sun at a focus), apogee (Earth at a focus), or apastron (star at a focus). The orbital point that is closest to the central body is called the perihelion (Sun), perigee (Earth), or apastron (star).



The eccentricity of an ellipse can be defined as the ratio of the distance between the foci to the major axis of the ellipse. The eccentricity is zero for a circle. Of the planetary orbits, only Pluto has a large eccentricity.

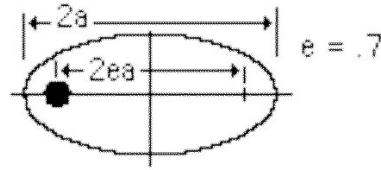
Examples:



Planetary orbit eccentricities

Mercury	.206	Saturn	.0556
Venus	.0068	Uranus	.0472
Earth	.0167	Neptune	.0086
Mars	.0934	Pluto	.25
Jupiter	.0485		

Johannes Kepler (1571-1630), the German assistant and successor to [Tycho Brahe](#), believed the Copernican Heliocentric Model of the Solar System from his twenties on, and was destined to bring about acceptance of this concept. That is, he believed the sun rather than the earth was the center of the planetary system.



The life-long question that concerned Kepler was the nature of the timing and motion of the celestial machinery, for he was convinced that simple mathematical relations could make sense of the planetary system. He saw the planetary system operating according to its own set of mathematical laws which was quite a radical idea for his time.

Kepler was a mathematician rather than an observer. Yet, Kepler was supplied with years of impeccable data by his employer Tycho Brahe who had carefully marked the position of Mars in relationship to the rest of the celestial map. Kepler rejected many ideas, such as circular orbits, because they did not fit Brahe's observations. In 1609, Johannes Kepler finally published his first two laws of planetary motion in a book entitled *New Astronomy*. A decade later (1619), his third law was published in *The Harmonies of the World*.

Kepler developed his empirical laws from Brahe's data on Mars: *"By the study of the orbit of Mars," he said, "we must either arrive at the secrets of astronomy or forever remain in ignorance of them."* However, in what proved to be a revolutionary step, Kepler then generalized saying that his laws applied to all the planets, including the Earth, without ever actually verifying that this was indeed true. Now we now know, they even apply to comets. Perhaps even beyond Kepler's dreams, the generalization of his laws predict and explain the motion of satellites orbiting the earth, moons orbiting planets, stars orbiting another star, etc. The expectation that the mathematical laws of science are universal is so readily accepted in our time that it is difficult to comprehend just how important Kepler's actions were to science.

Kepler's work put to rest any notion that planets move in perfectly circular orbits because nature has decreed that the heavenly bodies must show perfection in their movements. He also put to rest in the scientific community an ancient idea that there exists a mystical complex motion of planets that somehow governs our ways. Although Kepler never knew why planets move by the empirical relationships articulated in his three laws, he diligently sought a cause of which these three laws were the effect. As he stated, *"I am much occupied with the investigation of physical causes. My aim in this is to show that the celestial machine is rather a clockwork..."*.

Kepler vaguely sensed that bodies have a natural "magnetic" affinity for each other and guessed that the Sun has an attractive force. However, it remained for Newton, half a century later, to formulate a unified theory of motion invoking gravity as the cause of planetary motion.

Solution to U8a WS5 Prob.#1:

$$M = (4\pi^2 / G) (a^3 / T^2)$$

$$a \text{ (for S2)} = \frac{17 + 24(10)}{2} = \frac{257}{2} = 128.5 \text{ light-hours}$$

$$a = 128.5 \times 1.08 \times 10^{12} \text{ m} = 1.3878 \times 10^{14} \text{ m}$$

$$a^3 = 2.6729 \times 10^{42} \text{ m}^3$$

$$T = 15.2 \text{ years} = 15.2 \times 3.16 \times 10^7 \text{ sec} = 4.803 \times 10^8 \text{ sec}$$

$$T^2 = 2.307 \times 10^{17} \text{ sec}^2$$

$$\text{Where } G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

$$\frac{4\pi^2}{G} = \frac{39.48}{6.673 \times 10^{-11}} = 5.916 \times 10^{11}$$

$$\text{Therefore, } M = \frac{4\pi^2}{G} \frac{(a^3)}{(T^2)} = 5.916 \times 10^{11} \times \frac{2.6729 \times 10^{42}}{2.307 \times 10^{17}} = 6.854 \times 10^{36} \text{ kg}$$

Dividing the answer by one solar mass of $2 \times 10^{30} \text{ kg}$ yields a mass of the Central Black Hole of:

$$\frac{6.854 \times 10^{36}}{2 \times 10^{30}} = 3.43 \text{ million Solar Masses}$$

See http://www.space.com/scienceastronomy/mystery_monday_031124.html for discussion of latest Black Hole estimate.

For years scientists said the black hole contained about 2.6 million times the mass of the Sun. They now believe the figure is somewhere between 3.2 million and 4 million solar masses.